Letters to a Princess

Leonhard Euler

From: Todd Doucet
To: Gentle Reader
Subject: email to a German princess
Date: July 2017

I don't know why I keep getting these emails to a German princess. I try to send them back but it doesn't work. They are notably old and don't even come in any particular order. But they're interesting and enjoyable to read.

They're in French. I can fix that. So I'll do that and assemble them here, in the hope that they will get to where they need to go.

Thanks.

Lettres à une princesse d'Allemagne sur divers sujets de physique et de philosophie, L'académie impériale des sciences, Saint Petersburg, 1768, 1772. Numbers 343, 344, and 417 in the Eneström index.

From: Leonhard Euler
To: Your Highness
Subject: distance
Date: Saturday, 19 April 1760

Madame,

As the hope of being able to continue my instruction in Geometry to Your Highness seems to be again receded, which gives me a very sensible chagrin, I would like to be able to supply it in writing, as much as the nature of the subjects permits. I will make this into an essay, explaining to Your Highness the right way to think about size, while including in it both the smallest and the largest distances that we are presently discovering in the world.

And to begin, it is necessary to fix a certain measure proportionate to our senses, one which we have a good idea about, as for example that of a foot. Once this length is established and put before our eyes, it can help us to understand all lengths, both the largest and the smallest; of the former, by determining how many feet they include; of the latter, by determining what part of a foot they take up. For by having an idea of a foot, we also have an idea of its half, of its quarter, of its twelfth part (called an inch), of its hundredth part, and of its thousandth, which is so tiny that it nearly escapes our view.

But it is necessary to consider that there are even animals which are no larger than this, which have limbs with flowing blood, and which manifestly contain still other living insects, which are in relation to them as small as they themselves are to us; from which it is understood that the smallest quantities are presently existing in the world, and that they are found divided into the most infinitely small parts. In this way, for example, although the ten-thousandth part of a foot would be invisible to us, it exceeds the size of an entire animal, and should seem quite large to it, were it to have some awareness.

But let us move on from these tiny quantities, where our spirit is getting lost, to the largest. Your Highness knows the length of a *mille*. There are 18 milles between here and Magdebourg. A mille is taken to be 24,000 feet, and we use them to measure distances between places on earth, in order to avoid the excessively-large numbers we would get by using the foot.

Knowing that a mille is 24,000 feet, when we say that Magdebourg is separated from Berlin by 18 milles, we get a clearer idea than if we were to say that this distance is 432,000 feet, because this number almost overwhelms our understanding. Similarly, we will get a good idea of the size of the entire earth when we will know that the circumference of the earth is 5400 milles.

Now, since the earth has the shape of a globe, the diameter of this globe works out to about 1720 milles, which gives us a good idea of the diameter of the earth, and this in turn is used to measure the largest distances that we find in the heavens.

Among the celestial bodies, the moon is the one which is the closest to us, its distance from earth being only about 30 diameters of earth, which makes 51,600 milles, or even 1,238,400,000 feet. But the first measure, 30 diameters of earth, is the clearest one.

The sun is about 300 times more distant than the moon, and therefore its distance of 9000 diameters of earth gives us a clearer idea than if we were to express it in milles or even in feet.

Your Highness knows that the earth revolves around the sun in the span of one year, and that the sun remains at rest. Now, there are, besides the earth, still five other similar bodies, which likewise revolve around the sun, but at either smaller distances, as Mercury and Venus, or at greater distances, as Mars, Jupiter, and Saturn, and these are called Planets.

All the other stars that we see, except for the comets, are called *fixed*, and their distances are incomparably larger than the distance to the sun. Their distances from us are without doubt very unequal, which results in some of them seeming larger than others.

But the closest one is certainly more than 5000 times farther than the sun, and therefore its distance exceeds 45,000,000 diameters of earth, and in milles it would be 77,400,000,000, and finally this number multiplied by 24,000 will give this prodigious distance expressed in feet.

This is still only the distance to the fixed stars nearest us; and the farthest ones which we see will be well more than 100 times further still.

However, one imagines that all these stars, taken together, constitute only a very small part of the entire universe, with respect to which these terrible distances are not any larger than a grain of sand in relation to the earth.

All this immensity is the work of the Almighty, who equally governs the largest bodies and the smallest, and who directs the success of the arms to which we are engaged.

From: Leonhard Euler
To: Your Highness
Subject: speed
Date: Tuesday, 22 April 1760

In the hope that Your Highness will accept the continuation of my instruction, a sample of which I have taken the liberty to present to her by the previous post, I will develop the idea of speed, which is a particular kind of size, being susceptible to more or to less.

When a thing is transported, or passes from one place to another, we attribute to it a speed. We imagine a post rider and a foot messenger, going from Berlin to Magdebourg, and we conceive in each of them a certain speed, but we say that the speed of the first is greater than that of the second.

It is a matter, then, of examining what the difference between these two speeds we see entails. It is not the route, which is the same for the rider and the messenger; but the difference is found clearly in the time that the one or the other takes to do the same route.

The speed of the rider is therefore greater, since he takes less time to go through the route from Berlin to Magdebourg; and the speed of the messenger is less, since he takes more time to do the same route. From this it is clear that in order to form a good idea of speed, it is necessary to have in view two kinds of quantity at once, namely the route which is traversed and the time elapsed.

Thus a body which covers in the same time twice the distance, will have twice the speed; and if it covers in the same time a route three times longer, its speed is deemed three times greater, and so on. We will know, then, the speed of a body, when we know the route it travels and the time it takes.

In this way, in order to know my walking speed when I go to Lytzow, I observed that I take 120 steps each minute. Now, one of my steps takes two and a half feet. So my speed is such that I traverse in one minute a path of 300 feet, and in one hour I cover a path sixty times greater, or 18,000 feet. This is still not a mille, which is 24,000 feet and would take an hour and 20 minutes. So if I wanted to walk from here to Magdebourg, it would take me precisely 24 hours.

So this gives a good idea of the speed which I am capable of walking; and from this it is easily understood what it means to have either a greater or a lesser speed. Thus, if a post rider went from here to Magdebourg in 12 hours, his speed would be two times greater than mine; and if he went in 8 hours, his speed would be three times greater.

We observe a very great difference among speeds in the world. A tortoise gives us an example of a very small speed; if it goes only one foot per minute, its speed would be 300 times smaller than mine, since I go 300 feet in one minute.

Now, we also know about speeds which are very much greater. The speed of the wind is very variable: a mediocre wind goes 10 feet in one second, or 600 feet per minute, so it goes two times faster than me. A wind which covers 20 feet in a second, or 1200 per minute, is already reasonably strong. A wind which goes 50 feet per second is extremely strong, although its speed is only 10 times greater than mine, and it would take 2 hours and 24 minutes for it to blow from here to Magdebourg.

After this comes the speed of sound, which goes 1000 feet in one second, and therefore 60,000 feet per minute. So it is 200 times greater than the speed at which I walk. If one fired a cannon in Magdebourg, and it were possible that the sound carry to Berlin, it would arrive after only 7 minutes time.

A cannonball moves at nearly the same speed; but when a bigger charge is used, it is reckoned that it should be able to travel 2000 feet in one second, or 120,000 feet per minute. This speed seems to us prodigious, although it exceeds by only 400 times my speed walking to Lytzow, and this is also the greatest speed that we glimpse down here on earth.

But there are in the heavens much greater speeds, though the movements seem to us most tranquil. Your Highness knows that the earth revolves around its axis in the span of 24 hours; so at the equator this speed reaches 5400 milles in 24 hours, while I could only traverse 18 milles of it. So this speed is 300 times greater than mine, and therefore less than the greatest speed of a cannonball.

Now, the earth moves around the sun in the span of one year, and with this speed it travels 128,250 milles in 24 hours; so this speed is 18 times more rapid than that of a cannonball.

The greatest speed that we know is without doubt the speed of light, which travels 2,000,000 milles each minute, and which exceeds the speed of a cannonball by 400,000 times.

From: Leonhard Euler
To: Your Highness
Subject: sound and its speed
Date: Saturday, 26 April 1760

The explanations on the diverse degrees of speed which I have taken the liberty to present to Your Highness leads me to the consideration of sound, or of arbitrary noise in general; I have observed that some time always elapses before it reaches our ears, and that these times increase as the place where the sound is produced gets farther away from us; so that in order to be communicated over a distance of 1000 feet, it takes one second of time.

When a cannon is fired, those who are separated from it hear the noise only some time after they have seen the flame of the powder. Those who are a mille away, or 24,000 feet, do not hear the noise for 24 seconds after catching sight of the flame.

Your Highness will have also often observed that the clap of thunder reaches our ears only some time after the flash of lightning; and from this we can judge the distance from us to the place where the thunder is generated.

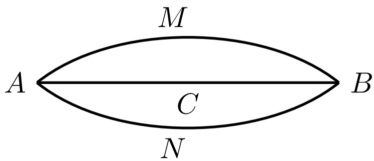
If we observe, for example, that 20 seconds elapses between the flash of lightning and the thunder, we can conclude that the source of the thunder is 20 times one thousand feet distant from us, by counting for each second of time one thousand feet of distance.

This nice property leads us to the question, what does sound consist of? Whether the nature of sound resembles that of odor? Whether sound is given off in the same way from a sounding body, that a flower gives off its odor by filling the air with subtle exhalations suitable to excite our sense of smell?

One could have had thought this in antiquity, but at present we are well convinced that when a bell is struck, nothing at all leaves from it to be transported to our ears, or better, that all bodies which emit sound do not lose any of their substance.

One has only to look at a bell when it is struck, or a string when it is plucked, to see that the body is then found trembling or shaking, and all its parts are agitated. And all bodies which are susceptible to such a shaking of their parts produce a sound too.

In a string, whenever it is not too thin, one can see these shakes or vibrations by which the string ACB goes alternately between the configurations AMB and ANB, which I have drawn much more exaggerated than actually happens.



Next it is necessary to observe that these vibrations put the neighboring air into a similar vibration, which is successively communicated to the parts of the air farther away, until they come to hit the organ of our ear.

It is thus the air which receives such vibrations, then transports the sound to our ears; from which it is clear that the perception of sound is nothing other than our ears being struck by the shaking found in the air, which is communicated to our organ of hearing; and when we are hearing the sound of a plucked string, our ears are receiving from it as many strikes as the string has vibrated during that same time.

In this way, when the string makes 100 vibrations in a second, the ear receives from it also 100 strikes per second, and the perception of these strikes is what is called a sound.

When these strikes follow equally one after another, or when their intervals are all equal, the sound is regular and such as one demands from music; but when these strikes succeed each other unequally, or when the intervals between them are unequal, it results in an irregular noise altogether unsuitable for music.

When I consider a little more carefully the musical sounds whose vibrations occur equally, I observe first that when the vibrations (as well as the strikes upon the ear), are stronger or weaker, it does not result in a difference in the sound other than that it becomes louder or softer; and this is the difference that musicians indicate by the words *forte* and *piano*.

But a much more essential difference is when the vibrations are more rapid or less, or when more or fewer of them happen per second. In this way, when one string achieves 100 vibrations per second, and another string 200 vibrations per second, their sounds will be essentially different from each other; the first will be flatter or lower, and the second sharper or higher.

So note the true difference between the flat and the sharp sounds, upon which all of music revolves, which teaches the mixing of sounds which differ among themselves in relation to flat and sharp, yet are joined together in such a way that an agreeable harmony results from them.

Now, as for the flat sounds, they have fewer vibrations in a given time compared to the sharp sounds, and each sound on the clavichord contains a certain and determined number of vibrations which it achieves in one second. In this way, the sound indicated by the letter C makes about 100 vibrations per second, and the sound indicated by the letter $\overline{\overline{\overline{c}}}$ makes 1600 vibrations per second.

So a string which trembles 100 times per second will give precisely the sound C, and if it were to tremble only 50 times, the sound would be lower or flatter still.

Now, in regard to our ears, there are limits beyond which the sounds are no longer perceptible. It seems that we can no longer hear a sound which makes fewer than 20 vibrations per second, because it is too low, nor can we hear a sound which would make more than 4000 vibrations per second, because it is too high.

From: Leonhard Euler
To: Your Highness
Subject: consonance and dissonance
Date: Tuesday, 29 April 1760

Your Highness just interrupted the thread of my thoughts in a very gracious manner . . . .

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So it is with a heart filled with gratitude that I return to my subject. I have observed that while hearing a simple musical sound, our ear is hit by a series of strikes equally separated from one another, of which the frequency, or number produced in a certain time, causes the difference which governs the flat and sharp sounds, so that the smaller the number of vibrations or strikes produced in a certain time (such as one second), the more the sound is deemed flat; and the larger this number, the more the sound is sharp. The sensation of a simple musical sound can then be compared with a series of equally-spaced points, like this:

. . . .

If the intervals between these points are larger or smaller, the sound represented by them will be respectively flatter or sharper. There is no doubt too that the sensation of a simple sound resembles, or is analogous to, the view provided by such a series of equally-spaced points; and by means of this one can visually represent the same thing that the ears sense when hearing a sound. If the distances between the points were unequal, with the points arranged chaotically, this would be the representation of a confused sound contrary to harmony.

Taking this to be the case, let us consider what effect two sounds made at the same time must produce in the ear. First, it is clear that if these two sounds are equal—if each contains the same number of vibrations during the same time—the ear will be affected by it in the same way as from a single sound, and in music one says that these two sounds are in unison, which is the simplest *chord*, a chord being the name given to the mixture of two or more sounds that one hears at the same time.

But if the two sounds differ with respect to their flatness or sharpness, one will discern a combination of two series of strikes, and in each of these the intervals are equally spaced, but the intervals are longer in one than the other, with the longer intervals corresponding to the flatter sound, and the shorter intervals to the sharper sound.

Such a mixture, or chord, of two sounds can be represented visually by two series of points, arranged on lines *ab* and *cd*. In order to have a proper understanding of these two series, it is first necessary to discern its structure, or what comes to the same thing, the relation between the intervals in each line:

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     1     2     3     4     5     6     7     8     9     10    11
a    .     .     .     .     .     .     .     .     .     .     .    b
c    .    .    .    .    .    .    .    .    .    .    .    .         d
     1    2    3    4    5    6    7    8    9    10   11   12
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Here I have labeled the points for both lines, and have placed the bottom label 1 under the top label 1. But the labels for 2 will no longer be precisely one under the other, and still less for 3. But we see that the top label 11 is found precisely above the lower label for 12. From this, we know that the high sound achieves 12 vibrations while the lower makes only 11 vibrations.

But without the numbers being written there, the eyes would scarely discover this order, and it is even the same for the ears, which would discover with as much difficulty the order between the two sounds that I have represented by the two arrangements of points. But in this figure:

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.  .  .  .  .  .  .  .  .  .  .

.     .     .     .     .     .
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one sees at a glance that the upper line contains twice the number of points as the bottom one, and that the intervals in the lower line are twice as long as the intervals in the upper one.

This is without doubt the simplest case, after unison, where one can easily discover the order in these two series of points; and it is even the same with the two sounds represented by these two lines of points—one achieving precisely twice as many vibrations as the other—and the ear will easily discern this beautiful relationship between these two sounds, while in the preceding case the judgment is very difficult, if not impossible.

Now, when the ear easily discovers a relation governing the two sounds, their chord is called a *consonance*, and when this relation is difficult to discover, or even impossible, the chord is called a *dissonance*.

So the simplest consonance is where the sharp sound achieves precisely two times the number of vibrations as the flat sound. This consonance is called in music an *octave*, and everyone knows its force, and two sounds which differ precisely by one octave harmonize so well and resemble each other so much, that musicians even indicate them by the same letters. And so do we also see in the churches, that the women sing an octave higher than the men, yet consider themselves to be intoning the same notes.

Your Highness will easily assure herself of this truth on the clavichord, and will discern with pleasure the beautiful accord between all the sounds which differ by an octave, while two other arbitrary sounds do not ring as well.

From: Leonhard Euler
To: Your Highness
Subject: unison and octaves
Date: Saturday, 3 May 1760

Your Highness will have already observed that the chord which the musicians call the octave strikes the ear in such a distinctive way that the least aberration in it is readily noticed. Thus having intoned the sound indicated by F, one easily tunes the sound f, which is one octave higher, judging solely by ear; and if the string for the sound f is even a little bit too high or too low, the ear is immediately bothered by it: nothing is easier than to put it perfectly in tune.

Also do we see that everyone, when singing, easily passes from one sound to another, when it is one octave either higher or lower. But if it is necessary to go from the sound F to the sound d, for example, a mediocre singer will easily make a mistake if he is not aided by an instrument. Having fixed the sound F, it is nearly impossible to tune the sound d in one step.

So what is the reason for this difference, that it is so easy to tune the sound f to the sound F, but so difficult to tune the sound d from it? This reason is very clear by what I have had the honor to explain to Your Highness in my last remarks: it is that the sound F and the sound f make an octave—the number of vibrations of the sound f is precisely double of that of the sound F. To perceive this accord, it is only a matter of sensing the proportion of one–to–two which, just as it is immediately clear to the eye by the representation of points which I used previously, so it affects the ear in a similar way.

Your Highness will easily understand that the more a proportion is simple, or expressed by small numbers, the more it is presented distinctly to understanding, and excites in it a feeling of pleasure. Architects observe this maxim very carefully also, by employing everywhere in buildings proportions which are as simple as circumstances permit. In doors and windows, they ordinarily make the height two times larger than the width, and everywhere they try to employ proportions expressible by small numbers, since that pleases understanding.

So it is even the same in music, where the chords please only to the extent that the spirit discerns in them the proportion which governs the sounds, and this proportion is perceived more easily to the extent that it is expressible by small numbers.

Now, after the proportion of equality, which indicates two sounds are equal or in unison, the proportion of two–to–one is without doubt the simplest, and that furnishes the chord of the octave; and from this it is clear that this chord has a special place among consonances.

After this explanation of the chord or interval between two sounds that the musicians call an octave, let us consider several sounds, such as F, f, $\bar{f}$, $\bar{\bar{f}}$, and $\bar{\bar{\bar{f}}}$, in which each is an octave higher than the previous one. Since the intervals from F to f, from f to $\bar{f}$, from $\bar{f}$ to $\bar{\bar{f}}$, and from $\bar{\bar{f}}$ to $\bar{\bar{\bar{f}}}$ are each one octave, the interval from F to $\bar{f}$ will be a double octave, the interval from F to $\bar{\bar{f}}$ a triple octave, and the interval from F to $\bar{\bar{\bar{f}}}$ a quadruple octave. Now, while the sound F is yielding one vibration, the sound f is yielding two, the sound $\bar{f}$ is yielding four, the sound $\bar{\bar{f}}$ eight, and the sound $\bar{\bar{\bar{f}}}$ sixteen; from which we see that just as one octave corresponds to 1–to–2, in the same way a double octave corresponds to 1–to–4, a triple octave 1–to–8, and a quadruple octave 1–to–16.

Now, since the proportion of 1–to–4 is no longer as simple 1–to–2 (it is not as apparent to the eye) a double octave is not perceived as easily as a simple octave; a triple octave is still less perceptible, and a quadruple octave still less. So when tuning a clavichord and the sound F is fixed first, it is not as easy to tune the double octave $\bar{f}$ as the simple f; and it is still more difficult to tune the triple octave $\bar{\bar{f}}$ and the quadruple $\bar{\bar{\bar{f}}}$, without first climbing through the intermediate octaves.

These chords are also included in the term consonance, and since the consonance of unison is the simplest, one can arrange them according to the following degrees:

1^{st} degree, unison, indicated by the proportion 1–to–1
2^{nd} degree, the octave, in the proportion of 1–to–2
3^{rd} degree, the double octave, in the proportion of 1–to–4
4^{th} degree, the triple octave, in the proportion of 1–to–8
5^{th} degree, the quadruple octave, in the proportion of 1–to–16
6^{th} degree, the quintuple octave, in the proportion of 1–to–32

and so on, inasmuch as the sounds are still perceptible. These are the chords, or consonances, which we have been led to know thus far; and we do not yet know about any other kinds of consonances, and still less of the dissonances, that are used in music.

But before going on to explain the former, I must add a remark about the name octave, given to the interval of two sounds in which one makes twice as many vibrations as the other. Your Highness sees the reason for it in the principal keys of the clavichord, which climb in seven steps before reaching the octave, as C, D, E, F, G, A, B, c, so that the key c is the eighth, counting C as the first. But this division depends on a certain kind of music, and the reason needs to be explained in what follows.

From: Leonhard Euler
To: Your Highness
Subject: other consonances
Date: Saturday, 3 May 1760

It can be said that the proportions 1–to–2, 1–to–4, 1–to–8, and 1–to–16 which we have considered up to now, and which encompass the nature of an octave—simple, double, triple, quadruple—draw their origin solely from the number 2, since 4 is two times two, and 8 is two times four, and 16 is two times 8.

By admitting, in this way, only the number *two* into music, we come to know only the chords or consonances that musicians call the octave—simple, double, triple—and since the number 2 supplies by its reduplication only the numbers 4, 8, 16, 32, 64—one being always double the other—all other numbers still remain unknown to us.

Now, if an instrument contained only octaves, as in the sounds C, c, $\bar{c}$, $\bar{\bar{c}}$, $\bar{\bar{\bar{c}}}$, and if all the others were excluded, it would not be able to produce any pleasant music at all, because of its excessive simplicity. So let us introduce, besides the number 2, also the number 3, and let us see which chords or which consonances result from it.

At the beginning, the proportion 1–to–3 introduces two sounds to us; one yields three times more vibrations than the other during the same time. This proportion is, without doubt, the easiest to understand after 1–to–2, and so it will furnish some most beautiful consonances, but of a nature altogether different from the octaves.

So let us suppose that, in the proportion 1–to–3, the number 1 corresponds to the C sound. Since the c sound is expressed by the number 2, the number 3 gives us a higher sound than c, but still lower than the $\bar{c}$ sound, which corresponds to the number 4. Now, the sound expressed by 3 is the one that musicians indicate by the letter g, and they call the interval from c to g a *fifth*, since on the clavichord keys, starting with c and ending with g, there are five of them: c, d, e, f, g.

Then if the number 1 gives the C sound, the number 2 gives c, the number 3 gives g, the number 4 gives the $\bar{c}$ sound; and since $\bar{g}$ is an octave above g, its number will be 2 times three, or 6, and then by climbing yet another octave, the $\bar{\bar{g}}$ sound will be two times larger, or 12.

Starting with C being indicated by 1, the numbers 2 and 3 lead us to all of these sounds:

$$\begin{array}{cccccccccccccccc} C & . & c & . & g & . & \bar{c} & . & \bar{g} & . & \bar{\bar{c}} & . & \bar{\bar{g}} & . & \bar{\bar{\bar{c}}} \\ 1 & . & 2 & . & 3 & . & 4 & . & 6 & . & 8 & . & 12 & . & 16 \end{array}$$

From this it is clear that the proportion 1–to–3 expresses an interval made up of an octave and a fifth, and that this interval, because of the simplicity of its numbers, must be, after the octave, the most perceptible to the ear.

Musicians count the fifth as second in rank among the consonances, and the ear is so agreeably affected by it that it is quite easy to tune a fifth. On the violin, the four strings climb by fifths in this way, the lowest being G, the second d, the third a, and the fourth $\bar{e}$; and every musician easily puts them in tune by ear. However a fifth does not tune as easily as an octave; but the fifth above the octave, as from C to $\bar{g}$, is expressed by the proportion 1–to–3, and is therefore more perceptible than a simple fifth, as from C to G, or one from c to g which is expressed by the proportion 2–to–3. And it is known from experience that, having fixed the C sound, it is easier to tune the superior fifth g from it, than the simple fifth G.

If unity had indicated to us the F sound, the number 3 would indicate the $\bar{c}$ sound, so that the sounds and their corresponding numbers would be

$$\begin{array}{ccccccccccccc} F & . & f & . & \bar{c} & . & \bar{f} & . & \bar{\bar{c}} & . & \bar{\bar{f}} & . & \bar{\bar{\bar{c}}} \\ 1 & . & 2 & . & 3 & . & 4 & . & 6 & . & 8 & . & 12 \end{array}$$

where the interval from f to $\bar{c}$ is a fifth having the proportion 2–to–3; the intervals from $\bar{f}$ to $\bar{\bar{c}}$ and from $\bar{\bar{f}}$ to $\bar{\bar{\bar{c}}}$ are also fifths, since the proportions 4–to–6 and 8–to–12 are the same as 2–to–3. (Because if two yards cost 3 écus, 4 yards would cost 6 écus, and 8 yards 12 écus.) From the above, we discover another interval, this one having the proportion of 3–to–4, from $\bar{c}$ to $\bar{f}$, and therefore also from c to f, or from C to F. Musicians call it a *fourth*, and since it is expressed by larger numbers, it is far from being as pleasing as the fifth, and even less so than the octave.

The number 3 gave us these new chords or consonances, the fifth and the fourth. So before using other numbers, let us take the number 3 three more times, to get 9, which will give a higher sound than 3 or $\bar{c}$ by an octave and a fifth. The $\bar{\bar{c}}$ sound is the octave from $\bar{c}$, and $\bar{\bar{g}}$ is the fifth from $\bar{\bar{c}}$, and so the number 9 gives the sound $\bar{\bar{g}}$, and $\bar{\bar{c}}\,.\,\bar{\bar{\bar{f}}}\,.\,\bar{\bar{g}}\,.\,\bar{\bar{\bar{c}}}$ will be indicated by 6, 8, 9, 12. By taking these sounds in the lower octaves, the proportions will remain the same, and we will have

$$\begin{array}{ccccccccccccccccccccccccc} C & . & F & . & G & . & c & . & f & . & g & . & \bar{c} & . & \bar{f} & . & \bar{g} & . & \bar{\bar{c}} & . & \bar{\bar{f}} & . & \bar{\bar{g}} & . & \bar{\bar{\bar{c}}} \\ 6 & . & 8 & . & 9 & . & 12 & . & 16 & . & 18 & . & 24 & . & 32 & . & 36 & . & 48 & . & 64 & . & 72 & . & 96 \end{array}$$

from which we come to know some new intervals. The first is the one from F to G, having the proportion 8–to–9, which musicians call a *second*, and also a *whole tone*. The other is from G to f, having the proportion 9–to–16, which is called a *seventh*, and is a second, or a whole tone, smaller than an octave. These proportions are indeed expressed by considerably large numbers, and so the intervals are not counted among the consonances, and musicians call them *dissonances*.

If we take the number 9 three more times, to get 27, this number will indicate a higher tone than $\bar{c}$, and precisely a fifth higher than g. So this will be the tone $\bar{d}$, and its octave $\bar{\bar{d}}$ will correspond to 2 times 27, or 54, and the double octave $\bar{\bar{\bar{d}}}$ will correspond to 2 times 54, or 108. Let us represent these tones starting from several octaves lower, in the following manner:

$$\begin{array}{ccccccccccccccc} C & . & D & . & F & . & G & . & c & . & d & . & f & . & g \\ 24 & . & 27 & . & 32 & . & 36 & . & 48 & . & 54 & . & 64 & . & 72 \end{array}$$

$$\begin{array}{ccccccccccccccccc} \bar{c} & . & \bar{d} & . & \bar{f} & . & \bar{g} & . & \bar{\bar{c}} & . & \bar{\bar{d}} & . & \bar{\bar{f}} & . & \bar{\bar{g}} & . & \bar{\bar{\bar{c}}} \\ 96 & . & 108 & . & 128 & . & 144 & . & 192 & . & 216 & . & 256 & . & 288 & . & 384 \end{array}$$

where we discover that the interval D to F has the proportion 27–to–32, and the interval F to d has the proportion 32–to–54, or let us take half of that, 16–to-27. The first interval, above, is called a *minor third* and the other a *major sixth*.

We could triple the number 27 again, but music doesn't extend that far, and we limit ourselves to 27, which we got by multiplying 3 by itself three times.

The other musical tones that we still lack are introduced by the number 5, which I will develop in the next letter.

From: Leonhard Euler
To: Your Highness
Subject: the twelve tones of the clavichord
Date: Saturday, 3 May 1760

The matter which I am taking the liberty to discuss with Your Highness is so dry that I have grounds to fear that it will soon bore her. But to get it over with, I am sending three letters today at the same time, in order to finish, in one stroke, this almost unappetizing subject.

My intention was to put before the eyes of Your Highness the true origin of the sounds used in music, which is almost totally unknown among musicians. For it is not theory which has led them to the knowledge of all these sounds. They are, rather, indebted to a hidden force of true harmony, which has worked so efficaciously upon the ears that the latter have, so to speak, been forced to receive the tones currently in use, though their correct form is not yet decided upon.

The principles of harmony are in the end reduced to some numbers, as I have had the honor to make plain, and I have observed that the number 2 furnishes only the octaves, so that having, for example, fixed the tone F, we have been led to the sounds f, $\bar{f}$, $\bar{\bar{f}}$, $\bar{\bar{\bar{f}}}$. Next the number 3 supplies the tones C, $\bar{c}$, $\bar{\bar{c}}$, $\bar{\bar{\bar{c}}}$, which differ from the above by a fifth; and the repetition of this same number 3 supplies more fifths beyond these first, which are G, $\bar{g}$, $\bar{\bar{g}}$, $\bar{\bar{\bar{g}}}$, and finally the third repetition of this number 3 adds to it the tones D, $\bar{d}$, $\bar{\bar{d}}$, $\bar{\bar{\bar{d}}}$.

The principles of harmony are linked to simplicity, which seems not to permit pushing the repetition of the number 3 any further, and so up to here we have only the following tones in each octave

$$\begin{array}{ccccccccc} F & . & G & . & c & . & d & . & f \\ 16 & . & 18 & . & 24 & . & 27 & . & 32 \end{array}$$

which certainly does not allow a lot of musical variety. But let us also introduce the number 5, and see what the tone will be which yields five vibrations while the tone F is making only one.

The f tone makes two during the same time, and the $\bar{f}$ tone makes four, and the $\bar{\bar{c}}$ tone makes six. The tone in question is therefore between $\bar{f}$ and $\bar{\bar{c}}$, and that is the one musicians indicate by the letter $\bar{a}$, whose chord with $\bar{f}$ is called a *major third*, and is found to make a most pleasant consonance, having the proportion of these small numbers 4–to–5. Furthermore, this $\bar{a}$ tone with the $\bar{\bar{c}}$ tone makes a chord having proportion 5–to–6, which is almost as pleasant as the above, and is also called a *minor third*, like the one we have already spoken of which has proportion 27–to–32: the difference is almost imperceptible to the ear.

This same number 5, when applied to the other tones G, c, d, will similarly give their major thirds taken in the second octave above, that is to say, the sounds $\bar{b}$, $\bar{e}$, and $\bar{\bar{f}}_s$ which, when moved to the first octave, will then give us these sounds with their numbers:

$$\begin{array}{ccccccccccccccccc} F & . & F_s & . & G & . & A & . & B & . & c & . & d & . & e & . & f \\ 128 & . & 135 & . & 144 & . & 160 & . & 180 & . & 192 & . & 216 & . & 240 & . & 256 \end{array}$$

Remove the F_s, and you will have the principal keys of the clavichord which, according to the ancients, constitute the kind called *diatonic*, and which results from the number 2, from the number 3 repeated three times, and from the number 5. By allowing only these tones, one is in a position to compose very beautiful and very varied melodies, whose beauty is uniquely based on the simplicity of the numbers which furnish these tones.

Finally, by applying for the second time the number 5, it will furnish the thirds of the four new tones A, E, B, F_s that we just found, and so we will get the sounds C_s, G_s, D_s, and A_s, so that the octave is now filled with 12 tones, precisely the same ones received in music. All these tones draw their origin from the three numbers 2, 3, and 5, by replicating 2 as many times as the octaves require; but for the 3, we only replicate it three times, and for the number 5, only twice.

So let us see, below, all the tones of the first octave, expressed by the numbers which follow, where we see the composition of each of the numbers in terms of 2, 3, and 5.

C	2 . 2 . 2 . 2 . 2 . 2 . 2 . 3	384	Difference
C_s	2 . 2 . 2 . 2 . 5 . 5	400	16
D	2 . 2 . 2 . 2 . 3 . 3 . 3	432	32
D_s	2 . 3 . 3 . 3 . 5	450	18
E	2 . 2 . 2 . 2 . 2 . 3 . 5	480	30
F	2 . 2 . 2 . 2 . 2 . 2 . 2 . 2 . 2	512	32
F_s	2 . 2 . 3 . 3 . 3 . 5	540	28
G	2 . 2 . 2 . 2 . 2 . 2 . 3 . 3	576	36
G_s	2 . 2 . 2 . 3 . 5 . 5	600	24
A	2 . 2 . 2 . 2 . 2 . 2 . 2 . 5	640	40
A_s	3 . 3 . 3 . 5 . 5	675	35
B	2 . 2 . 2 . 2 . 3 . 3 . 5	720	45
c	2 . 2 . 2 . 2 . 2 . 2 . 2 . 2 . 3	768	48

While the sound C yields 384 vibrations, the sound C_s yields 400 in the same time, and the others as many as the adjoining numbers indicate. Thus the sound c will yield, in the same time, 768, which is precisely double the number 384. And for the octaves which follow, one has only to multiply these numbers by 2, or by 4, or by 8. In this way, the sound $\bar{c}$ will yield 2 times 768 or 1536 vibrations, the sound $\bar{\bar{c}}$ will yield 2 times 1536 or 3072 vibrations, and the sound $\bar{\bar{\bar{c}}}$ will yield 2 times 3072 or 6144 vibrations.

To understand the formation of the sounds from these three numbers 2, 3, and 5, it is necessary to note that the points put between the numbers mean multiplication. Thus for the tone F_s the expression 2 . 2 . 3 . 3 . 3 . 5 means 2 times 2 times 2 times 3 times 3 times 5. Now 2 times 2 is 4, and 4 times 3 is 12, and 12 times 3 is 36, and 36 times 3 is 108, and 108 times 5 is 540.

We see by the above that the differences between these tones are not all equal, and that some are greater and others smaller; this is also what true harmony demands. But since the inequality is not considerable, all these differences are commonly regarded as equal, and the step between each tone and the next is called a *semitone*; and one sees that the octave is, in this way, divided into 12 semitones.

Several musicians indeed make them actually equal, though this is contrary to the principles of harmony: for in this fashion no fifth, nor any third, is correct, and the effect is even the same as if these tones were not well tuned. They agree too, that it is necessary to renounce the correctness of these chords in order to obtain the advantage of equality of all the semitones, so that the transposition of one tone to another arbitrary one changes nothing in the melodies. However they admit among themselves that when the same piece is played in the tone C or a semitone higher, C_s, it changes considerably in nature, from which it is clear that all the semitones are not effectively equal, though the musicians try hard to render them such, because true harmony opposes the execution of this design, which is contrary to itself.

So this is the true origin of the tones which are in use today, and which are drawn from the numbers 2, 3, and 5.

If one wanted to further introduce the number 7, the number of tones in an octave would become larger, and all music would be carried to a higher degree. But it is here that mathematics abandons harmony to the music.

From: Leonhard Euler
To: Your Highness
Subject: pleasure in beautiful music
Date: Tuesday, 6 May 1760

It is a question as important as it is interesting: why does beautiful music excite in us a feeling of pleasure? Scholars are very divided on this.

There are those who claim that it is just a totally bizarre thing, that the pleasure music causes is not based on any reason at all, seeing that the same music can be to the taste of some, but disliked by others. But far from settling the question, it becomes rather more complicated. For we would want to know the reason why the same piece of music can produce such different effects, since we must agree that nothing happens for no reason.

Others say that the pleasure we feel when listening to beautiful music consists in the perception of the order which governs it. This thought seems at first glance to be well founded enough and deserves to be examined more carefully.

Music contains two kinds of objects where some order finds a place. The first relates to the difference between tones, inasmuch as they are high or low, sharp or flat. Your Highness will remember that this difference is comprised of the number of vibrations that each tone makes in the same time. This difference which is found between the speed of the vibrations of all the tones is what is properly called harmony.

Thus two tones which differ by an octave excite the feeling of the proportion 1–to–2, a fifth, the proportion 2–to–3, and a major third, the proportion 4–to–5. One understands the order found in a given harmony, then, when one knows all the proportions which hold between the tones which make up the harmony. It is the judgment of the ear which leads to this knowledge. This judgment is more or less final, so it is clear why the same harmony is perceived by some, but not at all by others, especially when the proportions between the tones are expressed by somewhat large numbers.

But music contains, besides harmony, yet another object susceptible to order, which is the measure by which one assigns to each tone a certain duration: and the perception of the measure consists in the knowledge of the duration of all the tones, and the proportions born of them, whether, for example, one tone lasts two times, three times, or four times longer than another.

The drum and the tympani provide for us music which has only measure, since all the tones are the same, and there isn't any harmony there; similarly there is also music that has only harmony, to the exclusion of measure. Choral is one such music, where all the tones are of the same duration; a perfect music contains both harmony and measure.

Now, whoever hears music and understands, by the judgment of the ear, all the proportions on which both harmony and measure are based, it is certain that he has the most perfect possible understanding of this music; while another who perceives these proportions only partly, or not at all, understands nothing of it, or has an imperfect understanding. But the pleasure around which our question revolves is still very different from this understanding I just spoke of, though one may confidently hold that music cannot produce pleasure unless one has an understanding of it. For the mere understanding of all the proportions which govern a piece of music, both regarding harmony and measure, is still not enough to excite the feeling of pleasure: something more is needed, and nobody has yet laid it all out. To convince yourself that the mere perception of all the proportions in a piece of music is not sufficient, one has only to consider a very simple musical piece, which only climbs by octaves, where the perception of proportions is certainly the easiest. However, it is a far stretch to say that this music causes pleasure, although one has the most perfect understanding of it. So we say that pleasure requires an understanding which is not too easy, but which requires some effort. This understanding must, so to speak, cost us something. But in my opinion that is still not enough. A dissonance whose proportion consists of larger numbers is harder to understand, however a series of dissonances taken without choice and without design will not please. It is necessary, then, that the composer has followed, in the composition, a certain plan or design which he has executed with true and perceptible proportions; and then when a connoisseur hears this piece, and he understands, besides the proportions, the very plan and design that the composer had in view, he will feel that satisfaction which is the pleasure that beautiful music can strike upon the intelligent ear. So this pleasure comes from what one divines, so to speak, of the views and feelings of the composer, whose execution, to the extent it is judged favorably, fills the spirit with a pleasant satisfaction. It is more or less a similar satisfaction that one feels when seeing a good pantomime, where one can divine by the gestures and actions the feelings and the discourse that they represent, and which execute, besides that, an attractive design. That riddle of the chimney sweep which so pleased Your Highness provides a good example for me as well. As soon as one divines the sense and recognizes that it is perfectly expressed in the telling of the riddle, one feels a great pleasure from it; although a flat or poorly-told riddle does not cause any pleasure.

So these are, in my opinion, the true principles at the base of all judgments about beauty in pieces of music. But this is only the opinion of a man who understands nothing of it at all, and who consequently must be ashamed to have been so bold as to discuss this subject with Your Highness.

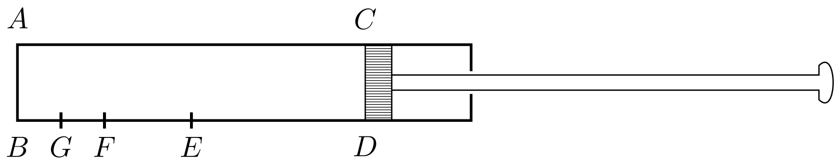
From: Leonhard Euler
To: Your Highness
Subject: the compression of air
Date: Saturday, 10 May 1760

The explanation of sound which I have had the honor to present to Your Highness leads me to a more particular consideration of the air, which is susceptible to a vibrational motion similar to what agitates audible bodies (strings, bells, etc.), and therefore transmits the shaking from those bodies to our ears.

We ask, then, what is the air? We do not initially perceive that this is actually matter. It seems that the space surrounding us—insofar as we do not see any perceptible substance there—does not contain any matter at all, since we do not feel anything there, and we can walk and move our limbs around, without encountering the slightest obstacle. But we only have to swat our hand very quickly to feel some resistance, and we will even feel the wind caused by such a rapid movement. Furthermore, wind is nothing other than air put into motion; and since the wind is capable of producing such astonishing effects, who could doubt that the air would be matter, and therefore also a substance? Because substance and matter mean the same thing.

We distinguish two kinds of substance: solids and fluids. It is evident that the air must be related to the class of fluids. It has several properties in common with water, but it is thinner and finer. It is known from experiment that air is about 800 times thinner and more rarefied than water; or if the air became 800 times thicker than it is now, it would obtain the same consistency as water.

A principle characteristic of air, which distinguishes it from other fluid matter, is that it allows itself to be compressed or reduced into a smaller space, which we show by this experiment. We take a metal or glass tube $ABCD$, sealed at the end AB, and open at the other end, where a piston is inserted which exactly fits the cavity of the tube. Then we push this piston in, and when it just reaches the location E, the air which at the beginning occupied the cavity $ABCD$, will then be reduced by half, and will consequently be twice as dense. If we push the piston even farther in, until F, halfway between B and E,



the air will be reduced into a space 4 times smaller; and if we continued pushing the piston all the way to G, where BG is half of BF, or one eighth of the entire length BD, the same air which was initially spread out over the entire cavity of the tube, would then be reduced to a space eight times smaller. If we were to continue in his way, reducing the air until it occupied a space 800 times smaller, we would obtain air 800 times denser or thicker than ordinary air. It would then be as dense and thick as water, which we are able to show by other experiments.

We recognize from this that air is a fluid matter that allows itself to be compressed, which means the same thing as to be reduced into less space. In this regard, air is a substance altogether different from water. If we were to fill the tube $ABCD$ with water and then put the piston there, it would be impossible to make the piston go in any farther. Whatever force we might use, the piston would remain absolutely still, and we would burst the tube before the water were reduced into even a slightly smaller space. So note this essential difference between air and water: water is not susceptible the slightest compression, whereas air can be compressed as much as desired.

The more we compress the air, the more it becomes denser or thicker. Thus air which occupied a certain space, when reduced or compressed into a space two times smaller, becomes twice as dense. When compressed into a space ten times smaller, it becomes ten times denser, and so on. I have already observed that if it became 800 times denser, it would have the same density as water, and would be as heavy, for weight increases in the same ratio as density.

Gold is the heaviest substance we know, and therefore also the densest. It has been found to be 19 times heavier than water, and a mass of gold shaped into a cube with length, width, and height each one foot, would weigh 19 times more than a similar mass of water. This mass of water weighs 70 livres, so the above mass of gold would weigh

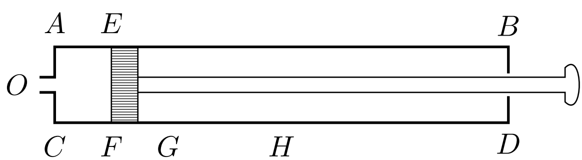
19 times 70, or 1330 livres. Therefore, if we could compress air until it were reduced to a space of 19 times 800 less, or 15200 times smaller, it would become as dense and as heavy as gold. But we are far from being able to compress air that much. Initially we can indeed advance the piston without effort, but the more it is advanced, the more effort is required to push it further; and before reaching the point of reducing it to a space ten times smaller, it would be necessary to use so much force to push the piston further that the tube would burst from it, unless it were very strong.

Not only would it take so much force to push the piston further, but it would take as much force to hold it, and as soon as we released the piston, the compressed air would push it back. The more the air is compressed, the more it tries to expand and to re-establish itself in its natural state. This is called the resiliency, or the elasticity, of air, which I propose to discuss with Your Highness by the next post.

From: Leonhard Euler
To: Your Highness
Subject: rarefaction and the elasticity of air
Date: Wednesday, 14 May 1760

Your Highness has just seen that air is a fluid matter, about 800 times thinner than water, so that if water could be spread out into a space that many times larger, and consequently became that many times thinner, it would be rather similar to the air we breathe. But air has a property it does not share with water: air can be compressed into a smaller space, where it becomes more concentrated, as I have had the honor to show by the previous post. But we discover yet another property of air which is no less remarkable: we can spread it out into a larger space, and by this means make it even thinner. This process is called the rarefaction of air, by which it becomes rarer, or more rarefied.

We take, as before, a tube $ABCD$, but in which there is a small hole O at the end AC, so that while inserting the piston toward F, air may escape through the hole, and it does not become concentrated.



The air which now occupies the cavity $ACEF$ will therefore be in its natural state, and then we stopper up the hole O. Next the piston is drawn back, and the air progressively spreads out into a larger space, so that when the piston will have been pulled back to G, the space CG is double the space CF, so the same air which was contained in the space $ACEF$ will now fill a space two times larger: it will therefore be two times less dense, or indeed twice as rarefied. When the piston is pulled back to H, the space CH is four times larger than CF, so the air will become four times more rarefied than it was at the beginning, being now spread out into a space four times as large.

And even when the piston would be pulled so far back that the space became 1000 times larger, the air would always spread out equally into this space, and would everywhere become 1000 times more rarefied. Here again air differs essentially from water: for if the cavity $ACEF$ were filled with water, however much the the piston were pulled back, the water would always occupy the same space as at the beginning, and the rest would remain empty. We learn from this that air is endowed with an intrinsic force to spread itself out more and more, which it exercises not only when it is concentrated, but also when it is rarefied. In whatever state of concentration or rarefaction the air may be, it tries to expand into a larger space, and it immediately spreads out as soon as it encounters no obstacle. This force making it spread out is called the resilience, or the elasticity, of air, and it is known from experiments similar to what I just spoke of that this force is proportional to the density; which is to say that the more the air is concentrated, the more it tries to expand; and the more rarefied it is, the less it tries to expand.

I might be asked, perhaps, why the air presently found in my room does not escape through the door, given that it is endowed with a force to expand into a larger space? Your Highness will no doubt respond that this would unfailingly happen if the air outside were not trying just as hard to expand. And since these efforts of the air in the room to leave, and the air outside to enter, are equal, they cancel another, and the air remains still.

Now, if outside air had acquired by some chance a greater density, and therefore also a greater elasticity, some of it would enter the room, where the air would be compressed and also acquire a greater elasticity. This would last until the elasticity of the inside air became equal to that of the outside air.

In the same manner, if the air in the room suddenly became denser, and its elasticity greater than the outside air, then the air in the room would leave, and by losing its density, it would lose as much elasticity, until its elasticity equaled that of the outside air. Then the motion would cease, and the air in the room would be in equilibrium with the outside air.

So too in the open, the air will be calm only to the extent it has the same degree of elasticity as the air in the surrounding region; and as soon as the air in one region becomes more or less elastic than in the neighboring region, the equilibrium can no longer subsist; but where the elasticity is larger, the air will expand and slide into places where the elasticity is smaller: and the wind is the result of such a movement of air.

It follows from this that the air in given place sometimes has a higher elasticity, and sometimes a lesser elasticity. This variation is indicated by an instrument called a *barometer*, whose description merits a particular explanation.

For the present, I limit myself to that quality of air by which it is concentrated or rarefied, and observe that the more concentrated the air is, the more force it has to expand, or better, the more its elasticity becomes larger; and conversely the more rarefied it is, the more it loses its elasticity.

Physicists have invented a machine that can both concentrate and rarefy the air, called a *pneumatic machine*. It is used to do several quite surprising experiments, most of which are already known to Your Highness. I intend to speak only of a few, as necessary to clarify and explain the nature and properties of air, which contributing principally to our conservation, and even to the production of all our earth-provided necessities, well merits that we form a just idea of it.

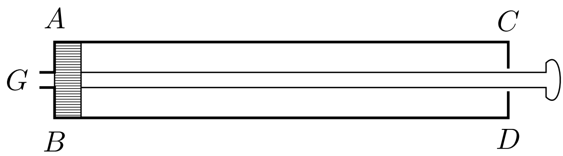
From: Leonhard Euler
To: Your Highness
Subject: the weight of air
Date: Saturday, 17 May 1760

I have had the honor to make clear to Your Highness that air is a fluid matter, endowed with this altogether singular property—that it can be compressed into less space, and that it expands into a larger one when obstacles are removed—so that the air is susceptible to both concentration and rarefaction. This property is comprised in the terms resilience or elasticity that are attributed to air, since the property is similar to that of a spring which can be tightened, and which springs back again when released.

But besides that, air also has a property common to all substances in general: that of gravity or heaviness, by which all bodies have a tendency to fall lower, and which makes them immediately descend when nothing supports them. Scholars are very divided and uncertain over the true cause of this force, but it is certain that this force actually exists. We are sure of it from daily experience. We even know the quantity of it, and we are in a position to measure it very exactly. For the weight of a body is nothing other than the force which pushes it lower; and since we can know and exactly measure the weight of each body, we know perfectly the effect of gravity (though the cause, this invisible force which acts on all bodies pushing push them lower, is absolutely unknown to us).

From the above we know that the more matter a body contains, the heavier it is. Thus gold or lead is heavier than wood or a feather, since it contains more matter in the same volume, or in the same space. So, because air is a form of matter which is so thin and so fine, its weight and its heaviness is also so small that it commonly escapes our senses. However there are experiments which convince us of it beyond doubt.

Your Highness has seen that we can rarefy the air in a vessel, or in a tube; and by means of the pneumatic machine it can be pushed so far that the air is completely removed from it, and the cavity of the vessel becomes completely empty. We take a tube $ABCD$ in which we have initially placed the piston so that it perfectly touches the



end, and so that there is no air remaining between the end and the piston. For better success, it is good to have a small hole in the end, G, through which the air may leave while we are pushing the piston towards the end. Then we block up the hole up with a stopper, making sure that there isn't any air hidden or compressed between the end and the piston. After this preparation we draw back the piston, and since the outside air cannot penetrate into the tube, we will have a perfect vacuum in the tube, between the end and the piston, which we can, by drawing the piston more and more, make as large as we want. By such a means we can remove the air from the cavity of a vessel; and when we weigh such a vessel emptied of air on a good balance, we find that it weighs less than if it were filled with air. From this we draw a very important conclusion: that the air contained in the hollow of the vessel increases its weight, and therefore the air itself has weight.

If the cavity of the vessel is so large that it can contain 800 livres of water, it is found, by this means, that the air which fills the same cavity weighs about one livre; from which we conclude that air is about 800 times less heavy than water. That must be understood to be the ordinary air which surrounds us and which we breathe; for Your Highness knows that air can be compressed by this art, forcing it into a lesser space, and by this means it likewise acquires additional heaviness. If the vessel I spoke of, above, which could contain 800 livres of water, were filled with air two times more compressed than ordinary air, it would weigh two livres more than if it were empty. If it were filled with air 800 times more compressed than ordinary air, it would weigh 800 livres more than if it were empty. Indeed, it would weigh as much as if it were filled with water.

Therefore, since air is a heavy substance (though in its natural state its weight is very small), it is endowed with a force to descend, and so it presses or weighs upon bodies found underneath which hinder its descent. It is for this reason that air which is higher up weighs upon air which is lower down, and the latter is found in a state of compression from all the weight of the air mass above. It follows that the air in our region has a certain degree of compression or density to which it is reduced by the weight of the air above it; and if the air above it were more or less heavy, our air would likewise become more or less compressed from it.

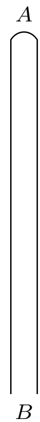
It is in this way that the lower air supports the weight of the upper air, and therefore, the higher we climb, on a tower or mountain, the more the air loses its density and becomes more rarefied; and by climbing ever higher, if it were possible, the air would finally completely run out, or would become so thin and so rarefied that it could not be perceived any longer. Conversely, when we descend into the deepest cave, the density of the air increases more and more, since there is a larger quantity of air above. If we made a hole to the center of the earth, the density of the air would increase more and more, acquiring that of water, and finally that of gold.

From: Leonhard Euler
To: Your Highness
Subject: the atmosphere and the barometer
Date: Tuesday, 20 May 1760

Having made clear that air is a fluid matter, compressible and heavy, I observe that the whole earth is everywhere surrounded by such air, which is called the atmosphere. It is also impossible that any region of the earth be devoid of air, and that there not be any air above the region at all, or that there be a perfect vacuum in the region: for the air from neighboring regions is compressed by the weight of the air above, and consequently it is continually trying to expand, and would suddenly spread out to the said region, and would fill the empty space. Thus the atmosphere fills all space around the earth, and in all places the air below supports the weight of the air above, and is compressed by it.

When air is compressed, its elasticity increases, and each degree of compression entails a certain degree of elasticity by which the air tries to spread itself out. Therefore the air is always compressed by the weight of the air above, up to the precise degree that its elasticity becomes equal to the force which is compressing it. So, though this air is compressed only from above, by virtue of its elasticity it tries to spread itself out in all directions, not only below, but also towards the sides. This is also the reason that the air in a room is compressed just the same as the outside air, which seemed quite paradoxical to some philosophers. For, they say, in a room, the air below is compressed only by the air found above it in the room, whereas the outside air is compressed by the full weight of the atmosphere, whose height is nearly immeasurable. But this doubt is immediately resolved by this property of air, which when being compressed, attempts to decompress in all directions; and the air in the room is immediately reduced to the same degree of compression and elasticity as the outside air. Thus, whether we find ourselves inside a room or outside, we would feel the same compression of air (it being well understood that these are at the same height, or the same distance from the center of the earth). For I have already observed that while ascending a high tower or mountain, the compression of the air is smaller, since the weight of the air above is then less.

Several phenomena verify to us beyond all doubt this compressed state of air. When we take a tube AB, closed at the end A, which has been been filled with water or another fluid matter, and we turn it upside down, so that the open end B is now on the bottom, then nothing flows out of it. The elasticity, or the compression of the air, which is pushing on the fluid at B, is supporting the fluid in the tube. But as soon as the tube is pierced at A, the fluid immediately falls, because the pressure of the air is then also acting at the top, and is pushing the water lower. We understand from this that as long as the tube is closed at the top, the force of the external air is what is supporting the water in the tube. If the tube were put into a vessel from which air had been withdrawn by a pneumatic machine, the water would immediately fall out.



The ancients, to whom this property of air was unknown, have said that nature supports the fluid in the tube because of the fear and even the horror that nature has of the vacuum. For, they say, if the fluid descended, there would be a vacuum at the top, because the air could not find a way to get into that space. So, according to them, it was fear of the vacuum which kept the fluid from falling out. Nowadays, it is certain that the force of the air supports the weight of the fluid in the tube. Since this force has a definite quantity, this effect would not be able exceed a certain point. It is found that if the tube AB, when filled with water, is more than 33 feet long, the water does not remain suspended there any longer, but it sinks far enough so that it stays in the tube only to a height of 33 feet, and above that there remains a true vacuum. Thus the force of the air can support the water in the tube only to a height of 33 feet; and since this same force supports weight of the entire atmosphere, we conclude that the atmosphere weighs as much as a column of water 33 feet tall. If instead of water, we take some mercury, which is 14 times heavier, the force of the air is only capable of supporting the mercury in the tube to a height of about 28 inches;

and if the tube is any higher, the mercury descends in it to a height appropriate to the pressure of the atmosphere, by allowing an empty space at the top of the tube.

Such a tube, stopped up at the top and open at the bottom, and filled with mercury, provides an instrument called a *barometer*; and this is how it is known that the atmosphere is not always equally heavy. For we know its true heaviness by the height of mercury in the barometer, which, becoming either greater or smaller, indicates that the air or the atmosphere has become either heavier or less heavy. This is the true indication of the barometer, and whenever it is rising or falling, this is a certain indication that the weight or the pressure of the atmosphere is increasing or decreasing, and this is what I had intended to present to Your Highness.

From: Leonhard Euler
To: Your Highness
Subject: air rifles and gunpowder
Date: Saturday, 24 May 1760

Having explained to Your Highness this singular property of air, that it can be forced into a smaller space, which is called the compression of air, we are in a position to report on several products, both from nature as well as from the art. I will begin by explaining the air rifle, not doubting that this instrument is well known to Your Highness.

The construction is more or less similar to that of the ordinary rifle, but instead of powder, compressed air is used to shoot the bullet. To understand how this works, we must observe that in order to compress air, we must employ as much additional force as whatever additional compression is needed.

When air is compressed, it tries to decompress; and these efforts are precisely equal to the force required to compress it to that point. So the more the air is compressed, the larger its effort to decompress; and if the air is forced to a density twice what is ordinary—which happens when air is pushed into a space two times smaller—the force with which it tries to decompress is equal to the pressure of a column of water 33 feet high.

Alternatively, Your Highness only has to imagine a large barrel of this height filled with water, and the water will certainly greatly press upon the bottom. If we made a hole in the bottom, the water would leave with great force. If we wanted to stop up this hole with a finger, we would feel this great force from the water, and every part of the bottom of the barrel is supporting a similar force. Now, a vessel which contains air two times denser than ordinary will feel a force precisely equal to that; and unless it were strong enough to support this force, it would burst. It is necessary, then, that the wall of this vessel be as strong as the bottom of the above-mentioned barrel.

If the air in this vessel were three times denser than ordinary, its force would be an additional one times greater, and the same as what the bottom of a barrel 66 feet high would support when filled with water. Your Highness will easily understand that this force will be very large; and it increases according to the same rule when the air is compressed 4 times, 5 times, or more, than ordinary.

Having established the preceding, there is at the bottom of the air rifle a completely sealed cavity, into which more and more air is forced, in order to compress the air into the greatest density that the forces used are capable of achieving; and by this means the air contained in this cavity will acquire a terrible force to escape; and when a hole is made there, air will immediately escape with this force. Such a hole is actually found there, which abuts the cavity of the tube, where a bullet is placed. This hole is stopped up well, but when one wants to shoot, a certain movement is made by which the hole opens for a moment, and the escaping air pushes the bullet forward with this great force, which we see when the bullet exits.

Each time one shoots, this hole remains open for only an instant, and therefore only a small quantity of air escapes from it, and enough air is left to shoot more several times. But each time the density of the air, and therefore also its force, diminishes. This is the reason that the subsequent shots are weaker than the initial ones, and why their force is finally lost entirely. If the hole mentioned above stayed open longer, more wind would escape from it, but for the most part unnecessarily; for this force only acts on the bullet as long as it is in the tube of the rifle: as soon as the bullet has left, it is of no use for the hole to remain open.

From this we will easily understand that if we could push the compression of the air much further, we could, with air rifles, produce the same effects as with ordinary rifles and canons. The effect of artillery is based on the same principle. Gunpowder is nothing other than matter which contains extremely compressed air in its pores. It is nature itself which has done for the pores the same things that we do when compressing air; but nature carries out this compression to a much higher degree. It is only a matter of opening these small cavities where this compressed air is held, in order to free the air and allow it to escape. This is done by means of fire, which breaks these small cavities, and this trapped air suddenly escapes with the greatest force, and pushes the bullets and the cannonballs in a manner altogether similar to what we have seen in the air rifle, but with much more force.

Behold then two quite surprising effects, which draw their origin from the compression of air, with the sole difference that in the first, the compression has been executed by art, and in the second, by nature itself.

We see here, as everywhere, that the operations of nature are infinitely superior to those which human skill is capable of producing; and we find in all places the most dazzling subjects to admire the power and the wisdom of nature's author.

From: Leonhard Euler
To: Your Highness
Subject: pyrometers and thermometers
Date: Saturday, 31 May 1760

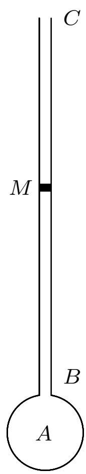
Besides the qualities of air that I have had the honor to relate to Your Highness, there is yet another very remarkable one, which air has in common with all substances, without even excluding the solids: the changes produced by the cold and heat. It has been generally observed that all substances, when heated, become larger. A bar of iron, when it is hot, is a little longer and a little thicker than when it is cold. There is an instrument called a *pyrometer*, which is constructed so that it indicates visually the slightest lengthening or shortening experienced by a bar to which it is applied.

Your Highness knows that in a watch, some wheels turn quite slowly, while the motion of others is very rapid, though these latter are nevertheless produced by the slow movements of the former. It is in this way that, by a kind of clockwork, one can make a nearly imperceptible change result in a quite considerable one, and this is what happens in this pyrometer instrument I just spoke of.

A bar of iron is placed there, or whichever other material is desired, and when the bar becomes even slightly longer or shorter, there is an indicator, like in a watch, which is deflected a very considerable amount. When this instrument is used on a bar of iron, or some other material, and a lamp is placed underneath to heat it, the indicator immediately starts to move, and climbs as the bar gets longer. The hotter it gets, the longer the bar grows. But when the lamp is extinguished and the bar is allowed to cool, the indicator moves in the opposite direction, indicating that the bar is becoming shorter again. However, this change is so small that we would be hard pressed to perceive it without the aid of this instrument.

We see this variation yet again in the pendulum clock. The pendulum is put there to moderate the motion, so that if the pendulum is lengthened, the clock runs slower, and if the pendulum is shortened, the clock runs faster. It is noted that in the high heat all these clocks run too slow, and in the extreme cold they run too fast, which is a clear indication that the pendulum is becoming longer in the heat, and shorter in the cold. Such variability caused by the heat and cold occurs in all substances; but the variability differs greatly according to the nature of the matter the substance is made up of, and some of them are much more sensitive to it than others.

In fluid substances, this variability is especially sensitive. To convince ourselves of this, we take a glass tube BC joined at the end B to a hollow ball A, and we fill it with whatever liquid we may up to the level, for example, M. Then when we heat the ball A, the liquid will climb from M towards C, and when the the ball becomes cold, the liquid descends down towards B. From this we see very clearly that the same liquid occupies a greater space when hot, and a smaller space when cold. We also see that this variation must be more sensitive when the ball is large and the tube narrow; for if the entire mass of liquid increases or decreases one part in a thousand, this thousandth part will occupy a larger space in the tube when the tube is narrower.



Such an instrument is, in turn, most useful to indicate to us various degrees of hot and cold; for if, in this instrument, the liquid is climbing or descending, that is a certain indication that the heat is increasing or decreasing. This instrument, which is called a *thermometer*, is used to show us changes in hot and cold. This instrument is totally different from the barometer, which shows us the weight of the air, or rather the force compressing the air down here. This distinction is all the more necessary because the barometer and the thermometer ordinarily resemble each other greatly, both being glass tubes fill with mercury. But their construction and the principles they are based on are totally different.

This same quality in which all substances are expanded by heat, and contracted by cold, also applies to the air, and this to a most remarkable degree. I intend to speak of it at greater length in the next post.

From: Leonhard Euler
To: Your Highness
Subject: changes that heat and cold produce in the atmosphere
Date: Saturday, 31 May 1760

Heat and cold produce the same effects on air as they do on all other substances. Air is rarefied by heat, and concentrated by cold. Now, by what I have had the honor to explain to Your Highness, a certain quantity of air is not limited to occupy a certain space, like all other substances; but by its nature air always tries to expand further, and also expands in fact as soon as it encounters no obstacle to oppose its outward expansion. This property is called the elasticity of air.

In this way, if air is trapped in a vessel, it tries to break out of the vessel; and this effort is greater insofar as the air is more concentrated in the vessel. From this we have drawn this rule: that the elasticity of air is proportional to its density; so that if the air is two times denser, its elasticity is also two times greater, and, in general each degree of density corresponds to a certain degree of elasticity. But now it must be observed that this rule is only true to the extent that the air is retaining the same amount of heat. As soon as the air becomes hotter, it acquires a greater force to expand than what would come from its density; and the cold produces an opposite effect in it by reducing its expansive force. So to know the true elasticity of an air mass, it does not suffice to know its density. It is also necessary to know the degree of heat that comes with it.

To put this into a better light, let us imagine that two rooms are sealed up but communicate with each other via a door, and that the same degree of heat prevails in the two rooms. Then the air must have the same density in both rooms. For if the air were denser and consequently more elastic in one room, some of the air would escape from that room and enter the other, until the densities in the two rooms became the same. But now let us suppose that one room becomes hotter than the other. The air in that room will acquire a greater elasticity from the heat, and will in fact spread itself out, and by entering the other room, it will reduce the air in that room into a lesser space, until the elasticity in that other room is brought to the same degree. While this is happening, there will be a wind passing by the door from the hot room into the cold, and when the equilibrium is re-established, the air will be more rarefied in the hot room and denser in the cold one. However, the elasticity of the air in both rooms will be the same. From this it is clear that two air masses with different densities can have the same elasticity, in particular when one is hotter than the other; and under this circumstance, it can happen that two air masses with the same density be endowed with various degrees of elasticity.

What I just said about two rooms can be applied to two regions of the earth, from which we understand that when one region becomes hotter than the other, air must necessarily flow from one to the other, resulting in wind. Here, then, is a very fertile source of winds (though there may be others too) which consist in the various degrees of heat prevailing in different regions of the earth. And it can also be shown that the air around the earth cannot be entirely at rest, unless all the air at each given height has not only the same density, but also the same degree of heat. And if there were no wind anywhere on the surface of the earth, one could surely conclude from it that the air would also be equally dense and hot at equal heights. Now, as this never happens, there must absolutely always be some wind, at least in some regions.

But these winds are for the most part found only on the surface of the earth, and the higher we rise the less violent the winds are. On the highest mountains, almost no more wind is observed, and a perpetual calm prevails there; from which it cannot be doubted that at even greater heights the air must remain always at rest. It follows that in regions so elevated, there will always prevail, around the entire earth, the same degrees of density and of heat. For if it were hotter in one place than in another, the air could not be at rest there, but instead there would be a wind. So since there is no wind in these elevated regions, the degree of heat in these regions must necessarily be, everywhere and always, the same. This is undoubtedly a most surprising paradox, having seen the great variations in heat and cold we experience here below, during the course of a year, and even from day to day, not to mention different *climates*, that is to say, the unbearable heat of the equator, and the extreme ice under the poles of the earth.

However, experience itself confirms the truth of this great paradox. In the high Swiss mountains, the snow and ice lasts throughout the summer and the winter, and on the *Cordilleras*, which are high Peruvian mountains in America situated at the equator, the snow and the ice are inalterable, and there is a prevailing cold as extreme as in the polar regions. The height of these mountains is not yet a German mille, or 24,000 feet, from which it can be confidently concluded that if we could fly to a height of 24,000 feet above the earth, we would find at that height, always and everywhere, the same degree of cold, and even an excessive cold. At that height, we would not observe any difference, neither during summer nor in winter, nor near the equator nor at the poles.

At this height and above, the state of the atmosphere is everywhere and always the same, and the variations between hot and cold only occur down below, near the surface of the earth. It is only here, down below, that the effect of the sun's rays becomes perceptible. Your Highness will undoubtedly be curious to learn the reason, and this will be the subject to which I will apply myself, in the next post.

From: Leonhard Euler
To: Your Highness
Subject: coldness on mountaintops and in caves
Date: Tuesday, 3 June 1760

It is quite a strange phenomenon that anywhere on earth, when one climbs to a great height like 24,000 feet (imagining this were possible) we experience the same degree of cold, while here at the surface the variations in heat are so considerable, not only in relation to various climates, but also in relation to the same place according to the different seasons of the year. This variation at low elevations is undoubtedly caused by the sun, and it would seem that its influence would have to be the same at high elevations and low ones, especially when we consider that a height of 24000 feet, or a mille, is absolutely nothing in relation to the distance from the sun (which is around 30 million milles), though this height seems quite large to us, and exceeds even the highest clouds. So this is a most important concern, which we should try to resolve.

To this end, I first observe that the rays of the sun heat a substance only to the extent that the substance does not allow the rays to pass freely through it. Your Highness knows these substances are called *transparent*, *pellucid*, and *diaphanous*, and we can see other objects through them. These substances are glass, crystal, diamond, water and several other liquids, though some are more transparent than others.

When such a transparent substance is exposed to sunlight, it does not become as heated as other non-transparent substances do, such as wood, iron, etc. Substances such as these which are not transparent are called *opaque*. Thus a burning lens lets the rays from the sun pass through it and burns an opaque body, and yet the lens itself does not get hot. Also, when water is exposed to sunlight, it becomes a little hotter only insofar as it is not perfectly transparent; and when we see that the water towards the edge of a river is somewhat heated by the sun, it is because the bottom, being an opaque body, is heated by the rays that pass through the water. Now, a hot substance always heats those which neighbor it, and therefore the water I just spoke of is heated by the bottom of the river. But if the water is very deep, so that the rays of the sun cannot penetrate all the way to the bottom, we do not feel hardly any heat there, even though the sun may shine brightly.

Now, air is a very transparent substance, even to a higher degree than glass or water; from which it follows that air cannot be heated by the sun, since the rays freely pass through it. All the heat that we often feel in the air is given to it by opaque bodies that have been heated by the sun's rays; and if it were possible to annihilate all these bodies, the air would experience hardly any change in its temperature caused by the sun's rays: it would remain equally cold, whether it were exposed to the sun or not. However, the air here near the surface is not perfectly transparent. Sometimes it is even so filled with vapor that it almost entirely loses its transparency, giving us fog. When the air is found in such a state, the sun's rays are absorbed more by it, and they can immediately heat the air. But such vapors do not climb very high, and at a height of 24,000 feet and above, the air is so thin and so pure that it is perfectly transparent, and therefore the sun's rays cannot immediately produce any effect there. This air is also too far away from terrestrial bodies for them to be able to communicate their heat to that air; such communication cannot go very far. From the preceding, Your Highness will easily understand that in the most elevated regions above the earth's surface, the sun's rays cannot produce any effect, and therefore there must always prevail there the same degree of cold, since the sun hasn't any effect there, and since the heat of the terrestrial bodies cannot be communicated there.

It is more or less the same on the highest mountains, where it is always colder than in the plains and valleys. The town of Quito, in Peru, is found almost on the equator, and judging from its location, the heat should be unbearable there. However the air there is quite temperate and not much different from that of Paris. Now, this town is situated at a great height above the true surface of the earth. When one goes there by sea, it is necessary to climb for several days, and so the terrain is as elevated as the highest mountains here, though it is still surrounded by the very high mountains called the Cordilleras. Because of this last circumstance, it would well seem that the air there would have to become as hot as on the surface of the earth, since it is everywhere touching opaque bodies on which the sun's rays fall. This objection is very strong, and there could not be any other reason than that the air in Quito is very elevated and therefore must be very much thinner and less heavy than the air here (as the barometer there being several inches lower than here incontestably proves). Such air is not as susceptible to heating as much as coarser air is, since it cannot contain as much of the vapor and the other particles which normally float in the air; and we know from experiment that well laden air is much more susceptible to heating.

I can add still another similar phenomenon, which is no less surprising, that in the deepest caves, or still lower if it were possible to travel there, there prevails, always and everywhere, the same degree of heat. The reason for it is more or less the same. As the sun's rays produce their effects only on the surface of the earth, from which it is communicated higher and well as lower, and as this communication is not able to penetrate very far, the very great depths are absolutely impervious to it, the same as with the very grand heights. I hope that this clarification will satisfy Your Highness.

From: Leonhard Euler
To: Your Highness
Subject: light rays and the systems of Descartes and Newton
Date: Saturday, 7 June 1760

Having spoken so much about the rays of the sun, which contain the source of all the heat and light we enjoy, Your Highness will no doubt ask, what are these rays of the sun? That is indisputably one of the most important questions in physics, and an infinity of phenomena depend on it. Everything regarding light and what makes objects visible to us is closely connected with this question. The ancient philosophers seem to be hardly worried about unraveling this question. Most were content to say that the sun is endowed with a quality to heat, illuminate, and shine.

But we have good reason to ask, in what does this quality consist? Is it something from the sun itself, something of its substance, that reaches us? Or alternately, would something cross over similar to a bell, whose sound reaches us without the least part of the bell being transported to our ears (as I have had the honor to show Your Highness by explaining the propagation and the perception of sound)?

Descartes, the first of the modern philosophers, supported this last opinion, and having filled the entire universe with a subtle matter made up of small globules, which he calls the second element, he puts the sun into a perpetual agitation which unceasingly strikes these globules, and these communicate their movements instantaneously throughout the universe. But since we have discovered that the rays from the sun do not instantly reach us, but that it takes them about 8 minutes to travel this great distance, the opinion of Descartes has been abandoned, and this is without mentioning the other great problems which accompany it.

Next, the great Newton has embraced the first opinion, and has argued that rays from the sun are really bodies from the sun, and that extremely fine particles are thrown or shot with this inconceivable speed, with which they are carried from the sun to us in about 8 minutes.

This opinion, which is that of most philosophers today, and especially of the English, is called the *system of emanation*, since it is believed that rays actually emanate from the sun, and also from other luminous bodies, like water emanates or jumps from a fountain. This opinion immediately seems very bold and shocking to reason; for if the sun continually ejected, in all directions, such rivers of luminous matter, with such a prodigious speed, it seems that the matter of the sun would soon be depleted by it; or at least it would be necessary to have observed, over the centuries, some diminution of it, which however is contrary to observation. Certainly a fountain which ejected streams of water in all directions would be soon depleted, especially as the speed of it would be greater; and so the prodigious speed of the rays would have to soon deplete the bodies from the sun.

However subtle we might imagine the particles forming the rays to be, we will gain nothing by it: the system remains equally objectionable. We cannot say that this emanation is not done from all around and in every direction, because in whatever place we may be, we see the entire sun, which incontestably proves that rays from all points of the sun are thrown towards this place. The case is then quite different from even that of a fountain which ejects streams of water in all directions. Here, there is only one place from which the stream leaves towards a given region, and each point would throw out only a single stream. But for the sun, each point on the surface throws out an infinity of streams which are spreading out in all directions. This single detail infinitely increases the amount of luminous matter that the sun must eject.

But there is yet another problem, which does not seem very small, which is that not only does the sun eject these rays, but so do all the stars. Then, since everywhere there would be rays from the sun and from the stars which would encounter each other, with what impetuosity must some collide with others? And how much must their direction be changed by it? A similar intersection must happen with all the luminous bodies we see at one time, however each appears distinct, without suffering the slightest disturbance from the others. This is a quite certain proof that several rays can pass through the same point without bothering each other, which seems irreconcilable with the system of emanation. Indeed, we only have to make it so that two streams of water intersect each other to immediately see how they interfere with each other's motion, from which we see that the motion of light rays is very essentially different from that of water jets, and in general from any matter which would be thrown out.

Next, by considering transparent bodies, through which rays pass freely in all directions, the partisans of this opinion are forced to say that these bodies contain pores arranged in straight lines, which go from each point on the surface out in all directions, since we cannot think of any line along which a sun's ray could not travel, and this with inconceivable speed, and even without hitting anything. Behold a body so riddled with holes, but which appears to us quite solid.

Finally, in order to see, it is necessary that the rays enter into our eyes, and that they traverse the substance with the same speed.

I believe that all these problems will sufficiently convince Your Highness that this system of emanation cannot in any way have any place in nature, and Your Highness will without doubt be quite astonished that this same system has been imagined by such a great man, and embraced by so many luminary philosophers. But Cicero has already observed that one cannot imagine anything so absurd that philosophers are not capable of supporting it. For me, I am too little of a philosopher to embrace this opinion.

From: Leonhard Euler
To: Your Highness
Subject: problems with the system of emanation
Date: Tuesday, 10 June 1760

However strange it might seem to Your Highness, this opinion of the great Newton that rays come from the sun by an actual emanation, it has however found such a wide approbation that almost nobody would dare doubt it. What has contributed to this the most is, without doubt, the great authority of this eminent English philosopher, who first discovered the true laws of motion for celestial bodies.

Now, this same discovery led him to the system of emanation. Descartes, in order to support his explanation, was forced to fill all space in the heavens with a subtle matter, through which all celestial bodies move totally freely. But we know that if a body moves through the air, it encounters a certain resistance. Newton concluded from this that however subtle one might suppose the matter of heaven to be, the planets would have to feel some resistance to their motion. But, he said, this motion is not subject to any resistance. From this it follows that the immense space of the heavens does not contain any matter. So there prevails everywhere a perfect void, and this is one of the principal dogmas of the Newtonian philosophy, that the immensity of the universe contains no matter at all in the spaces found between celestial bodies.

Given that, there would be between the sun and us, or at least up to the earth's atmosphere, a perfect void. Indeed the higher we climb, the thinner we find the air, to where it seems that it must finally be lost altogether. Now, if the space between the sun and the earth were absolutely empty, it would be impossible for the rays to come to us by means of communication as the sound from a bell is communicated to us by the motion of the air (so that if the air between the clock and us were annihilated, we would hear absolutely nothing, no matter how hard we struck the bell). Having then established a perfect vacuum between celestial bodies, there is no remaining opinion to embrace other than the one of emanation. And this reasoning forced Newton to maintain that the sun and similarly all other luminous bodies throw off actual rays, and that the rays are always a real part a luminous body, which is driven off with a terrible force. It would well be necessary that this force be terrible, to impart to the rays this inconceivable speed where they come from the sun to us in about 8 minutes time.

But let us now see whether this explanation can subsist with the principal view of Newton, which demands an absolutely empty space in the heavens, so that the planets do not encounter any resistance. Your Highness will easily judge that space in the heavens, far from remaining empty, will be filled with rays, not only from the sun, but also from all the other stars, and that these rays are traversing space in every place and in all directions, continually, and with the greatest rapidity. Then the celestial bodies which traverse these spaces, far from encountering a void there, will encounter the matter of the luminous bodies in a state of terrible agitation, by which the bodies must be much more troubled in their motion than if this same matter were at rest there.

So Newton, having been afraid that a subtle matter, such as the kind Descartes supposed, would trouble the motion of the planets, was led to a quite strange expedient, and one totally contrary to his proper intention; seeing that, by this means, the planets would have to suffer a disturbance infinitely more considerable. Here is a very sad example of human wisdom, which wanting to avoid a certain problem, often falls into greater absurdities.

I have already had the honor to reveal to Your Highness so many other insurmountable difficulties (of which the system of emanation is filled), and now we see that the principal, and indeed unique, reason which committed Newton to this opinion, is so contradictory in itself that it trips him up completely. All these reasons taken together could not let us hesitate for a moment to abandon this strange system of emanation of light, however great may be the authority of the philosopher who established it.

Newton was, without contradiction, one of the greatest geniuses who has ever existed, and his profound science and his penetration into the most hidden mysteries of nature will always remain the most brilliant object of our admiration and that of our posterity. But the wrong turns of this great man must serve to humble us, and to recognize the weakness of the human spirit, which when elevated to the highest degree to which men are capable, nevertheless often risks being precipitated into the grossest errors. If we are subject to such sad falls in our investigations on the phenomena of this visible world, which strikes our senses, how unhappy we would be if God had abandoned us to ourselves in regard to things which are invisible and which relate to our eternal salvation. On this important matter a revelation is absolutely necessary for us: we must profit from it with the greatest veneration; and when it gives us something which seems inconceivable to us, we have only to remember our weak spirit, which goes astray so easily even for visible things.

Every time that I see these strong minds, who criticize the truths of our religion, and even mock it with the most impertinent self-importance, I think: puny mortals, how much, and how many things upon which you reason so lightly, are more sublime and more elevated than the ones on which the great Newton went so grossly astray. I would hope that Your Highness would never forget this reflection. The occasions happen here only too often where we have good need of it.

From: Leonhard Euler
To: Your Highness
Subject: another system on the nature of rays and light
Date: Saturday, 14 June 1760

Your Highness has just seen that the system of emanation of rays is subject to insurmountable difficulties, and that the idea of a void which would occupy all the space between celestial bodies cannot occur in any way, since the very rays of light themselves would completely fill it up.

We are then forced reconcile two things. The first is that the space between celestial bodies is filled with a subtle matter, and the second is that rays are not an actual emanation from the sun and other luminous bodies, so that part of their substance is thrown from it, as Newton claimed. This subtle matter, which fills all the space in the heavens between celestial bodies, is called the *ether*, whose extreme subtlety cannot be doubted. In order for us to form a proper idea of it, we only have to consider air, which is a very subtle material here at the surface, and becomes more so when we climb higher, and finally it is lost, so to speak, entirely, or alternately it will be indistinguishable from the ether. So the ether is also a fluid matter like air, but incomparably more subtle and finer, since we know that celestial bodies travel through it freely without encountering any perceptible resistance from it. It undoubtedly has an elasticity, by which it tries to spread itself out in all directions and to penetrate into spaces which would be empty; so that if by some chance the ether were driven from some location, the neighboring ether would rush into that space in an instant, and the space would be filled with it again.

By virtue of this elasticity, the ether is found not only high above our atmosphere, but it penetrates it everywhere and also insinuates itself into the pores of all bodies here on the surface, so that it crosses these pores freely. Thus if we use the pneumatic machine to pump the air from a vessel, it is not necessary to believe that there would then be a vacuum; it is the ether, which by passing through the pores of the vessel, fills it in an instant. And when we fill a sufficiently long tube of glass with quicksilver and turn it over to make a barometer, we believe that we see above the quicksilver a void where there isn't any air, since the air cannot pass through the glass. But this void, which is one in appearance only, is certainly filled with ether which enters the vessel without difficulty.

It is by this subtlety and elasticity of ether that I will one day have the honor to explain to Your Highness all the surprising phenomena of electricity. It is even very likely that ether has a much greater elasticity than air, and that a multitude of effects in nature are produced by this force.

I do not even doubt that the compression of air in gunpowder is a work of this force of the elasticity of ether. And since we know by experiment that this air is almost a thousand times more concentrated than ordinary, and that, in this state, its elasticity is also as many times greater, it must be that the elasticity of ether is as much greater, and consequently a thousand times greater than ordinary air.

Having seen previously that air, through these qualities, becomes suitable to receive agitations or shaking from bodies which emit sound, and to spread them out in all directions, which is what the propagation of sounds consists of, it is very natural that ether could also, under similar circumstances, receive shaking and pass them in all directions over greater distances. As the shaking in the air gives us *sound*, what may well be given to us by this shaking of the ether? I believe that Your Highness will easily work it out: it is the light, or the rays. Thus it seems very certain that light is related to ether in the same way that sound is related to air, and that the rays of light are nothing other than shakes or vibrations transmitted by ether, nearly the same as sound consists in shakes or vibrations transmitted by air. So there is nothing which is actually coming from the sun towards us, no more than from a clock when its noise reaches our ears. In this system, there is no danger that the sun, by shining, is losing the least bit of its substance, no more than a ringing bell does.

What I say about the sun must also be understood to apply to all luminous bodies, such as a wax candle, a tallow candle, etc. Your Highness will no doubt object that these terrestrial lights are consumed only too obviously, and that unless they are tended and continuously fed, their light will soon be extinguished, from which it seems that the sun must be consumed as well, and that the parallel to a bell is very poorly used. But it is necessary to consider well that these fires, besides shining, throw out smoke and a multitude of exhalations, which must be carefully distinguished from the light rays which illuminate. The smoke and exhalations undoubtedly cause a considerable loss which must not be attributed to the light rays. If we could free them of the smoke and other exhalations, the sole quality of light would not cause any loss.

We can make mercury glow by a certain technique, as Your Highness remembers well having seen, and by this light the mercury loses absolutely nothing of its substance, from which we see that the light itself does not cause any loss in the luminous body; all its light being caused by a certain agitation, or an extremely quick shaking in its slightest particles, which is communicated to the neighboring ether, and is transmitted from there in all directions by the ether over the greatest distances, the same as a shaking bell communicates to the air a similar agitation.

The more we consider this parallel between bodies which emit sound and light, the more we will find it in conformance and agreement with experiment. By contrast, the system of emanation recedes even more as we want to apply it to phenomena.

From: Leonhard Euler
To: Your Highness
Subject: the propagation of light
Date: Tuesday, 17 June 1760

In regard to the propagation of light by the ether, it is done in a way similar to the propagation of sound by the air, and as a shaking caused in the particles of the air constitutes sound, in the same way a shaking caused in the particles of the ether constitutes light or light rays, so that *light is nothing other than an agitation or shaking caused in the particles of the ether*, which is found everywhere, due to its extreme subtlety which allows it to penetrate all bodies. However, these bodies modify the rays in different ways, according to whether they transmit or arrest the propagation of the shaking. I will speak of this at length later, but for now I am limiting myself to the propagation of rays in the ether itself, which fills the immense space between the sun and us, and in general between all celestial bodies. That is where the propagation is completely free.

The first thing that comes to mind here is the prodigious speed of light rays, which is around 900,000 times faster than the speed of sound, which still covers a distance of 1000 feet each second. This terrible speed would already suffice to overturn the system of emanation, but in this system it follows naturally from our principles, as Your Highness will see with plain satisfaction. These are the same principles that the propagation of sound in air is based on, which depend, on the one hand, on the density of the air, and on the other hand, its elasticity.

For this dependence, we are given to know that if the density of the air became less, sound would go faster in it, and if the elasticity of the air became greater, the sound would likewise go faster. So if the density became less, and at the same time the elasticity became greater, there would be a double reason for the speed of sound to increase.

Let us imagine, then, that the density of the air is diminished to the point that it became equal to the density of ether, and that the elasticity of the air is increased to the point where it likewise becomes equal to the elasticity of ether. Then we will not be surprised that the speed of sound would become several thousand times greater than it actually is. For Your Highness will remember that according to the first ideas we formed about the ether, this material must absolutely be incomparably less dense or more rarefied than air, and at the same time also incomparably more elastic. These two qualities both contribute equally to increase the speed of shaking. So now the prodigious speed of light is far from a shocking thing. It is instead in perfectly good agreement with our principles. The parallel between light and sound is in this regard so well established that we can confidently maintain that if the air became as subtle and at the same time as elastic as ether, the speed of sound would become as fast as light. So if asked why light moves with such prodigious speed, we will respond that the reason is the extreme subtlety of the ether in conjunction with its surprising elasticity, and that as long as the ether maintains this same degree of subtlety and elasticity, light must necessarily also pass with this same degree of speed.

We could not doubt that ether has, throughout the universe, the same subtlety and the same elasticity. For if the ether were more elastic in one place than in another, it would be carried there by spreading itself out more until the equilibrium were completely re-established.

Therefore the rays of the stars move as fast as those of the sun. But since the stars are much farther from us than the sun, it takes the rays as much additional time to reach us. However prodigious the distance to the sun might seem to us, where the rays nevertheless reach us in 8 minutes time, the distance to the fixed stars closest to us is still more than 400,000 times farther away from us than the sun. So a ray of light which leaves this star will take 400,000 times 8 minutes before reaching us. That is 53,333 hours, or 2,222 days, or around six years.

So when seeing a fixed star at night, and even the brightest, since that is probably the closest, the rays which enter the eyes of Your Highness in order to form a representation of this star, left the star six years ago, having used so a long time to reach us.f

And if it pleased God to create right now at the same distance a new fixed star, we would see it only after six years pass, since its rays could not reach us sooner. And if at the beginning of the world the stars had been created at more or less the same time as Adam, he would not have been able to see them until the end of six years, and even then those which are the closest. Because for the ones farther away, it would have been necessary for him to wait all the more time before seeing them. So if God had created at the same time stars more than a thousand times farther away, we would not have seen them yet, however bright they might be, since 6000 years have not yet passed since creation. The first prophet to the court of Brunswick, Mr. Jerusalem, has perfectly used this thought in one of his sermons, where the following passage is found:

> *Raise your thoughts from this world that you inhabit, towards all the bodies of the world which are above you. Take in the space there is between the farthest your eyes can see and the bodies whose light, perhaps since the beginning of their creation to now, has not yet reached us. The immensity of the kingdom of God permits this painting.*

I am very sure that Your Highness will be more edified from this passage than the whole audience of Mr. Jerusalem, to which this sublime thought will have been inconceivable, and I hope that this reflection will spark in Your Highness the curiosity to be instructed on the rest of the details about the true system of light, from which flows the theory of color and of all vision.

From: Leonhard Euler
To: Your Highness
Subject: gravity or heaviness, as a general property of bodies
Date: Saturday, 23 August 1760

After all that I have said above about light and its rays, I will have the honor to discuss with Your Highness a general property of all bodies we know about: that of gravity, or heaviness. We observe that all bodies, both solid and fluid, fall lower once they are no longer supported. When I take a rock in my hand and let it go, it falls to the earth and would fall even farther if there were a hole in the earth. Even as I write this, my paper would fall to the earth if it were not supported by my table. The same thing happens with all bodies we know about—there are none which do not fall to earth once they are no longer supported or held. The cause of this phenomenon or this tendency, which is found in all bodies, is called their gravity or their heaviness. When we say that all these bodies have gravity, we understand that they have a tendency to fall, and that they will indeed fall once we remove anything that is holding them up.

The ancients did not have a clear enough understanding of this property. They believed that there were also bodies which, by their nature, climb higher, as we see in smoke and vapors, which instead of descending, climb higher. They called these bodies *light*, to distinguish them from others which have a tendency to fall.

But in these later times, we have recognized that it is the air which pushes this material higher. For in a space emptied of air (which we make using the pneumatic machine), the smoke and its vapors descend just as well as a rock, from which it follows that these materials are also, by their nature, subject to gravity and they have heaviness, just like the others. When these materials climb in the air, the same thing is happening to them as when some wood is pushed under water, and despite its heaviness, it comes back up and floats on top as soon as I let it go. The reason is that the wood is less heavy than the water. This is a general rule, that a body will climb in a fluid which is heavier than itself. In a vase filled with quicksilver, if we throw in some pieces of iron, copper, silver, or even lead, they float on the top, and if they are submerged they will come back up by themselves. Only gold falls to the bottom, because it is heavier than the quicksilver. Therefore, as there are bodies which rise in water, or in another fluid, despite their gravity, and this by the sole reason that they are less heavy than the water or other fluid, it is not surprising that certain bodies which are less heavy than air, such as smoke or vapors, would rise there.

I have already had the honor make the observation to Your Highness that air itself is heavy, and that by its heaviness it supports the mercury in the barometer. In this way, when we say that all bodies are heavy it must be understood that all bodies, without exception, would fall to the bottom in a space void of air. I would even add that they would fall in that space void of air with an equal rapidity, for under a bell jar which has had the air pumped out of it, a coin and a feather would fall with equal speed. But I will speak more fully about this later.

One could object to this general property of bodies, because a bomb launched by a mortar does not initially fall to earth like a rock that I would let fall from my hand, but it rises high. But do we want to infer from this that the bomb has no heaviness? It is only too evident that it is the force of the gunpowder which pushes the bomb higher, without which it would surely fall in an instant. We even see that the bomb does not climb forever, but that once the force pushing it higher ceases, the bomb falls and in fact obliterates everything it encounters, which is a complete proof that it has heaviness. So when we say that all bodies have heaviness, we are not denying that they can be stopped or even thrown into the air. But that is done by forces outside the body, and it is still always true that for a body, whatever it may be, once it is left to itself and it is at rest or not moving, it will certainly fall as soon as it is no longer supported.

Under my room is a cellar, but my floor supports me and prevents me from falling in. If my floor were to suddenly crumble and the arch of my cellar collapsed at the same time, I would, without fail, soon fall into my cellar. This comes from the heaviness of my body, the same as all the other bodies that we know about. I say *that we know about* because perhaps there could be some bodies without heaviness, like the bodies of angels who have appeared in times past. Such a body would not fall, even when the floor would be removed from under it, and it would walk as easily in the air as down here on the surface of the earth. Except for these bodies that we do not know about, the general property of all those that we know about is that they have heaviness, by virtue of which they all have a tendency to fall, and actually do fall once nothing opposes the descent.

From: Leonhard Euler
To: Your Highness
Subject: specific gravity
Date: Monday, 25 August 1760

Your Highness has just seen that gravity is a general property of all bodies we know about, and that it consists in a tendency, by an invisible force, which pushes these bodies lower. Philosophers argue a lot whether it is actually a force acting invisibly on bodies and pushing them lower, or whether it is instead an internal quality contained in the very nature of all bodies, like a natural instinct which causes them to descend. This question comes down to this, whether the cause of heaviness is found in the very nature of each body, or whether it exists outside them, so that if they ran out of it, the bodies would cease being heavy? Or more simply still, we ask whether the cause of heaviness exists in the bodies or outside them?

Before entering into this dispute, it is necessary to examine more carefully all the circumstances accompanying the heaviness of bodies. First, I observe that when we support a body in order to prevent it from actually falling, as when we set a body on a table, this table experiences the same force with which the body would like to fall, and when we attach the body to a string that we hold suspended, the string is tightened by the force which is pushing the body down, that is to say, by its heaviness, so that if the string were not strong enough, it would break. From this we see that all bodies exert a certain force on the obstacles which are supporting them and preventing them from falling, and that this force is precisely the same as that which would make the body fall if it were free. When we set a rock on a table, the rock presses on the table. We only have to put a hand between the rock and the table, and we will feel this force which can even become big enough to crush the hand. This force is called the weight of the body, and it is clear that the weight and the heaviness of each body signifies the same thing, both indicating the force by which the body is pushed lower, whether this force exists in the body itself or outside it.

We have too clear of an idea of weight for it to be necessary for me to pause further. I will only observe that when two bodies are joined together, their weights are also added, so that the weight of the composite is equal to the sum of the weights of the parts. From this we see that the weights of bodies can be very different from each other. We even have a very sure way to compare the weights of bodies with each other, and to measure them exactly. This is done with the aid of a balance which has this property, that when the bodies placed into its two bowls have equal weight, the balance is found in equilibrium. To succeed in this comparison, here we establish a fixed measure, which is a certain weight, as for example one livre, and by means of a good balance, we can weigh any given body, and assign to each the number of livres that its weight contains. If a body is too big to be put in a bowl on the balance, we divide it, and having weighed each of the parts, we only have to add the weights together. In this manner, we could find the weight of an entire house, however large it might be.

Your highness will have already observed that a small piece of gold weighs as much as a much larger piece of wood; from which we see that the weight of a body is not always ruled by its size. A very small body can have a great weight, while another very large one could weight very little. Each body is therefore susceptible to two altogether different measures. By the first, we determine its size, also called its volume, and this measure concerns geometry, where we are taught how to measure the size of a body. But the second way of measuring a body, by which we determine its weight, is altogether different. And this is how we distinguish different materials that form a body.

Let Your Highness imagine several masses of different materials, all the same size, for example, each having the shape of a cube whose length, width, and height are each one foot. Such a volume, if it were gold, would weigh 1330 livres. If it were silver it would weigh 770 livres. If it were iron it would weigh 500 livres. If it were water, it would weigh only 70 livres. And if it were air it would only weigh a twelfth of a livre. Your Highness sees that the different materials which form a body make a very considerable difference with respect to its weight. To express this difference, certain terms are used which could seem ambiguous, if they are not well understood. Thus when it is said, for example, that gold is heavier than silver, it must not be understood to mean that a livre of gold is heavier than a livre of silver, because a livre, of any material whatsoever, is always a livre, and always has precisely the same weight. Instead, the sense is that having two pieces of the same size, one gold and the other silver, the weight of the gold will be greater than the weight of the silver. Just like when we say that gold is 19 times heavier than water, the sense is that having two equal volumes of them, one gold and the other water, the one with gold will weigh 19 times more than the one with water. In this manner of speaking, we are saying nothing about the absolute weights of these bodies. Instead, we are speaking of them only by comparison, always relating them to equal volumes. It does not even matter whether these volumes are large or small, provided they are equal.

From: Leonhard Euler
To: Your Highness
Subject: terms relating to weight
Date: Wednesday, 27 August 1760

Gravity, or heaviness, seems to us so essential to the nature of a body that it is nearly impossible for us to imagine the idea that a body would not possess heaviness. This quality likewise enters so generally into all our endeavors that we are everywhere having to take into consideration the heaviness or the weight of bodies. We ourselves, whether we are standing, or sitting, or laying down, continually sense the effect of the heaviness of our particular bodies. We would never fall if our bodies and all its parts were not heavy, or endowed with this inclination which carries them lower once they are no longer supported. Our very language is driven by this property of bodies, and we use the word *lower* for the direction towards which this inclination of bodies is directed. This word does not mean anything else, and if this inclination tended towards a different direction, we would call this other direction *lower*. Similarly, we call the direction opposite from this *higher*.

It is necessary to observe that when we let a body fall freely, it always descends in a straight line, and we say accordingly that it is directed lower. This line, also called *vertical*, is consequently always a straight line drawn from high to low. And if we imagine this line extended in height up to the sky, we call this point in the sky our *zenith*, which is an Arabic word and means the point of the sky which is directly above our head. From this, Your Highness understands what a vertical line is: it is the straight line along which a body falls once it is no longer supported.

When we attach a body to a string which is held firm at the other end, this string, when at rest, will be in a taut straight line, which will also be a vertical line. This is the way builders use a string weighted by a ball of lead, called a *plumb line*, when they erect a wall which must be vertical, so that it does not fall down.

All the floors of a house must be erected such that the vertical line is perpendicular to it. Then it is said that the floor is *horizontal*, from which Your Highness understands that a horizontal plane is always one which is perpendicular to the vertical line.

When we are standing in a perfect plain not obscured by any mountains, the extremities are called the *horizon*, which is a Greek word which means the limit of our view. This plain, then, is represented by a horizontal plane, the same as the surface of a lake. We also use another term to designate what is horizontal. We say that such a surface is *level*.

We also say that two points are level when the straight line which passes through the two points is horizontal, so that a vertical line, or a plumb line, is perpendicular to it. But two points are not level when the straight line drawn through these points is not horizontal. Then one of these two points is more elevated than the other. That is what happens with rivers, where the surface has an incline, for if it were horizontal, the river would be at rest and would not flow, since all rivers always flow towards the least elevated places. There are instruments to determine whether two points are at a level, or whether one is higher than the other, and by how much. Such an instrument is called simply a *level*, and the art of using it is called *leveling*.

If Your Highness wanted to have a straight line drawn from a point in her apartment in Berlin, to a given point in her apartment in Magdebourg, by using this instrument we could find whether this line is horizontal, or whether one of the two points is either more or less elevated than the other. I believe that the point in Berlin would be more elevated than the one in Magdebourg. I base this opinion on the course of the Spree, Havel, and Elbe rivers. Since the Spree flows into the Havel, the Havel must be lower than the Spree. And by the same reasoning the Elbe must be lower than the Havel. It follows that Berlin is more elevated than Magdebourg, that is to say at street level, for if a straight line were drawn from street level in Berlin to the top of the bell in Magdebourg Cathedral, perhaps this line would be horizontal.

From this Your Highness can also understand how useful the art of leveling is when it concerns directing water, for since water only flows from a more elevated place to a less elevated place, before digging a canal for the water to flow, we must be completely sure that one end is more elevated than the other, which we will know by the art of leveling.

While building even a town, we must arrange the streets so that they are inclined towards a side, so that the water runs off. It is different for buildings, where one wants the floors of the rooms to be perfectly level and not have any slant, because there is no water to make flow away, unless it is a stable, where the floors are given an incline.

Astronomers are also very attentive to the floors of their observatories, which must be perfectly level, in order to correspond to the true horizon that we see in the sky, which makes the vertical line mark its zenith.

From: Leonhard Euler
To: Your Highness
Subject: Further clarifications on the nature of the spirit
Date: Saturday, 10 January 1761

To better clarify what I just observed about the difference between body and spirit (for one cannot be too attentive to what constitutes this difference, which extends even so far that the spirit has nothing in common with the body, nor the body with the spirit), I am going to add the following additional reflections.

Extent, inertia, and impenetrability are the properties of bodies; spirit has neither extent, nor inertia, nor impenetrability. For extent, all philosophers agree that it can have no place in relation to the spirit. The thing is self-evident, since everything which has extent is also divisible, or better, one can conceive of its parts; now, a spirit is not susceptible to any division: one cannot conceive of a half or a third of a spirit. Each spirit is instead an entire being which excludes all division: so one cannot speak of a spirit having length, width, or depth. In a word, all which we conceive of extent must be excluded from the idea of spirit. From there, it seems that, since spirits have no size, they are similar to geometrical points, which again have neither length, nor width, nor depth. But would it be a good idea to represent a spirit as a point? The Scholastic philosophers were of this opinion, and represented spirits as infinitely small beings, similar to the finest specks of dust, but endowed with an inconceivable activity and agility, by which they were able to jump in an instant the greatest distances. Because of this extreme smallness, they maintained that millions of spirits could fit in the smallest space: they even put to question how many spirits could dance on the point of a needle. The followers of Wolff have more or less the same opinion. According to them, all bodies are composed of extremely small particles, stripped of all size; and they give them the name *monads*: so that one monad is a substance without any extent: or better, by dividing a body until one reaches particles so small that they are not susceptible to any further division, one reaches the Wolffian monads, which then differ from a very fine dust only in that the molecules of the powder are perhaps not small enough, and it would be necessary to divide them still further in order to obtain the true monads.

Now, according to Mr. Wolff, not only are bodies composed of monads, but also each spirit is nothing other than a monad; and even the Supreme Being—I nearly dare not write this—is again such a monad; which gives hardly a magnificent idea of God, or of spirits, or of our souls. I cannot conceive that my soul is only a being of the same nature as the ultimate particles of a body, or that it be only nearly a point. Still less does it seem to me to be supportable that several souls, taken and joined together, could form a body: for example a piece of paper with which one could light a pipe of tobacco. But the proponents of this opinion hold fast to this reasoning that, since a spirit has no extent, it must indeed be similar to a geometrical point. So it all comes down to examining whether this reasoning is solid or not.

I remark first that, since a spirit is a being of a nature altogether different from that of a body, one cannot even undertake questions which assume a size, and it would be absurd to ask how many feet or inches a spirit is long, or how many livres or ounces it weighs. These questions can be made only about things which have length or weight. It would also be absurd, while speaking about time, if one wanted to ask, for example, how many feet an hour would have in length, or how many livres it would weigh. I can always say that an hour is not equal to a line of 400 feet, or of 40 feet, or of one foot, neither of any other measure; but it does not follow that an hour is a geometrical point. An hour is of an altogether different nature, and questions do not apply to it which assume a length expressible by feet or by inches.

It is the same with a spirit. I can always confidently say that a spirit isn't ten feet, nor a hundred feet, nor any other number of feet, but from that it does not follow that a spirit must be a point; nor that an hour must be a point because it cannot be measured by feet or by inches. So a spirit is not a monad, or similar to the ultimate particles into which a body can be divided; and Your Highness will now understand very well that a spirit can have no extent, without for this reason being a point or a monad. It is necessary then to separate all ideas of extent from the idea of a spirit.

It would also be an absurd question to ask in what place a spirit exists; for, as soon as one attaches a spirit to a place, one assumes that it has extent. Nor can I say in what place an *hour* is to be found, although an hour is without doubt a thing: thus something can be without it being attached to a certain place. In the same way, I can say that my soul does not exist in my head, nor outside my head, nor in whatever place, without one being able to draw from it the consequence that my soul doesn't exist at all; neither of the present hour, which I can truly say exists neither in my head nor outside my head. So a spirit exists without it existing in a certain place; but if we reflect upon the power that a spirit can have to act on a certain body, this action is made without doubt in a certain place.

In this way, my soul does not exist in a certain place, but it acts in a certain place; and since God has the power to act on all bodies, it is in this regard that one says that God is everywhere, though his existence is not attached to any place.

From: Leonhard Euler
To: Your Highness
Subject: Continuation on the same subject, and reflections on the state of the soul after death
Date: Tuesday, 13 January 1761

Your Highness will find it quite strange, the opinion I just advanced, that the spirits, by virtue of their nature, are nowhere. By saying these words, I would risk being taken for a man who denies the existence of spirits, and consequently also that of God. But I have already made clear that a thing can exist and have reality without it being attached to any place. The weak example drawn from an hour removes the biggest difficulties, though there is still an infinite difference between an hour and a spirit.

This idea which I imagine about spirits seems to me infinitely more noble than the idea of those who regard spirits as geometrical points, and who include even God in this class. What could be more shocking than to confound all the spirits, and even God, with the smallest particles into which a body can be divided, and to line them up in the the same class with these feeble particles, which do not become nobler with the fancy name of *monad*?

Being in a certain place is an attribute which belongs only to corporeal things; and since spirits are of an altogether different nature, one cannot be surprised when it is said that spirits are not found in any place, or what amounts to the same thing, are nowhere; and after these clarifications I fear no reproach in this regard. It is by this means that I elevate the nature of spirits infinitely above that of bodies. All the spirits are thinking beings: reflecting, reasoning, deliberating, acting freely, and in a word living; while the bodies have no qualities save those of being extended, being susceptible to movement, and being impenetrable; from the latter, the following universal characteristic results, that each body remains in the same state as long as there is no danger of any penetration; and in the case where the bodies would be penetrated if they were to continue in their present state, their *impenetrability* even supplies the necessary forces to change their state as much as required to prevent all penetration. All the changes which happen in bodies consist of such things; all are merely passive, and all happen by necessity in conformance to the laws of motion. In bodies there is neither intelligence, nor volition, nor freedom; these are the eminent qualities of spirits, while bodies are not even susceptible to them.

It is also from the spirits that, in the corporeal world, the principle events and the good deeds find their origins; and this happens by the action and influence that the souls of men have on each of their bodies. Now, this power that each soul has on its body can only be regarded as a gift from God, who has established this marvelous bond between souls and bodies; and since my soul is found in such a bond with a certain small part of my body hidden in the brain, I can very well say that the seat of my soul is in the same place, though properly speaking my soul doesn't exist anywhere, and relates to this place only by virtue of its action and its power. It is also the influence of the soul on the body which constitutes its life, which lasts as long as this bond persists or as long as the organization of the body remains intact. Death is then nothing other than the destruction of this bond; subsequently the soul has no need to be transported elsewhere; for since it isn't anywhere, it is indifferent to all places; and consequently if it would please God to establish after my death a new bond between my soul and a body organized on the moon, I would be in an instant on the moon, without having made any voyage; and even if, right now, God additionally accorded to my soul a power over a body organized on the moon, I would be equally here and on the moon, and there would not be any contradiction in that. It is only a body which cannot be in two places at once; but for a spirit, which has no relation to a place by virtue of its nature, nothing prevents it from being able to simultaneously act on several bodies situated in places most distant from each other; and, in this regard one could very well say that it is found in all these places at once.

This provides a nice explanation to aid our understanding of how God is everywhere: it is that his power extends to all the universe and to all the bodies found in it. For this reason it seems to me that it would not be right to say that God exists everywhere, since the existence of a spirit does not relate to any place; rather it would be necessary to say that God is present everywhere, and this is also the language of Revelation.

Let us now compare this idea to that of the Wolffians, who, presenting God in the form of a point, attach him to a certain place, since, indeed, a point cannot be in several places at once; and how could one reconcile omnipresence with the idea of a point, and again the idea of a point with omnipotence?

Death being a dissolution of the union which subsists between the soul and the body during life, we can form some idea of the state of the soul after death. As the soul, during life, draws all its knowledge by means of the senses, being stripped by death of this relationship to the senses, it no longer learns anything about what passes in the material world; it reaches more or less the same state as a man who finds himself suddenly blind, deaf, mute, and deprived of the use of all the other senses. This man would keep the knowledge he had acquired by aid of the senses, and he would be well able to reflect; the proper actions he had committed especially could provide for him a broad subject matter; in the end the faculty of reasoning would remain entirely intact, since the body does not contribute to it in any way.

Sleep also provides us with a fine preview of this state, because in sleep the union between the soul and the body is in large part interrupted; though the soul cannot then cease being active and occupies itself with its reveries, which provides dreams. Ordinarily the dreams are most troubled by the remaining influence that the senses still have on the soul, and it is known from experience that the more this influence is abated, resulting in a very deep sleep, the more the dreams are regular and connected. In this way, after death we will find ourselves in a state of most perfect dreaming, which nothing will be capable of troubling any longer: these will be representations and reasoning perfectly well sustained. And that is, in my opinion, nearly all that we can say about it for sure.